Govt. Engineering College, Ajmer

Department of Civil Engineering

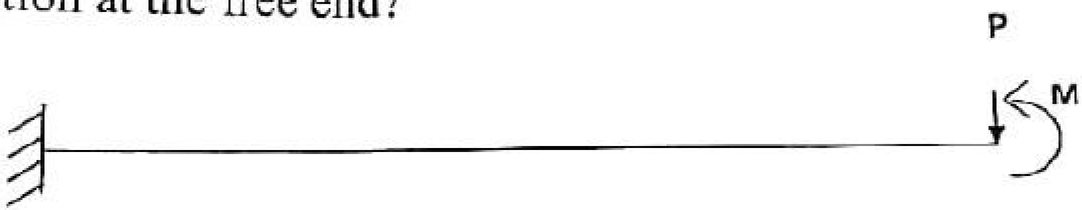
I[st] Mid Term test 2017-18

Time:-1hr. | MM 10

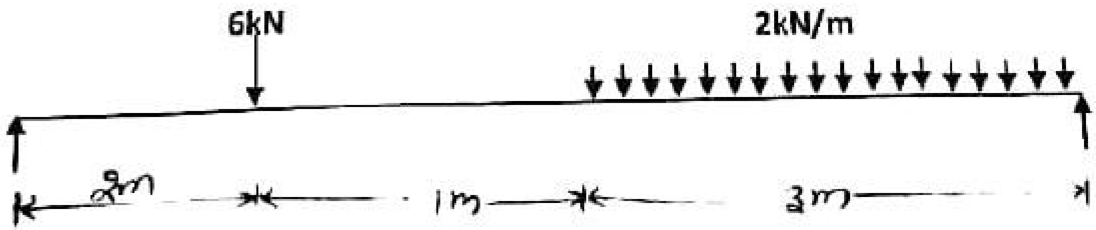
Subject:-SOM-II

Q.1. Explain area moment equations and also write sign conventions for area moment method? 2

Q 2. A cantilever of length 'L' is subjected to a concentrated load P and a couple M acting at a free end as shown in figure. Determine the slope and deflection at the free end? 4



Q 3. A beam AB of 6m span is simply supported at the ends and is loaded as shown in figure. Determine deflection at C and slope at end A? 4



In order to determine the deflection ... Mohr's theorems ... in the next article ... deviation which in turn can be found from the Mohr's theorems ... next article.

13.3. MOHR'S THEOREMS : AREA MOMENT EQUATIONS

(a) **Rotation** : Fig. 13.2 shows the elastic curve $AmnB$ of a beam. AB' is the tangent to the curve at A and BB'' is the tangent to the curve at B. Evidently, BB' is the deviation (d_B^A) of B with respect to tangent at A. The angle between the two tangents is θ_B^A. If θ_A is the rotation of tangent at A and θ_B is the rotation of tangent at B, we have, in general

$$\theta_B^A = \theta_B - \theta_A \qquad ...(13.2)$$

In Fig. 13.2, θ_B is negative (since the rotation is anticlockwise)

Hence $\theta_B^A = (-\theta_B) - \theta_A$

$$= -(\theta_B + \theta_A) \qquad ...(13.2a)$$

In order to find the values of θ_B^A and d_B^A, let us consider two sections m and n at a very small distance ds along the curve. After bending, the normal sections at m and n will subtend an angle $d\theta$ at the centre of curvature. Hence $ds = R \cdot d\theta$

and $$\frac{1}{R} = \left| \frac{d\theta}{ds} \right| \qquad ...(i)$$

The bars in the above expression indicate that we consider here only the numerical value of curvature. Regarding sign, B.M. is taken positive if it produces concavity. *Hence curvature is positive when the centre of curvature*

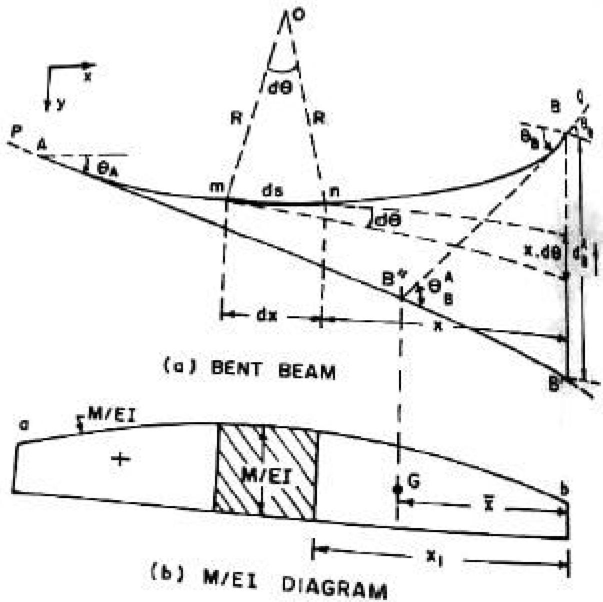


FIG. 13.2

... above the curve, as in Fig. 13.2. By close inspection of Fig. 12.1(*b*), we observe that for ... curvature, the angle θ decreases as the point *m* moves along the curve from point ... to *Q*. Thus we have proper sign for Eq. (*i*) which must be written as

$$\frac{1}{R} = -\frac{d\theta}{ds}$$

But

$$\frac{1}{R} = \frac{M}{EI}$$

Hence

$$d\theta = -\frac{M}{EI}ds$$

Since the curvature is very small, we can take *ds* equal to *dx*.

Hence

$$d\theta = -\frac{M}{EI}dx \quad \text{...(13.3)}$$

The term $\frac{M}{EI}dx$ has a simple geometric interpretation. Directly below the deflection curve is shown the $\frac{M}{EI}$ diagram. It should be noted that the $\frac{M}{EI}$ diagram will have the same shape as the *M*-diagram, only if EI is constant along the entire length of the beam. The ordinate of the (M/EI) diagram is equal to the ordinate of the B.M. diagram (*i.e.* M-diagram) divided by the *flexural* rigidity *EI* at that point. The term $\frac{M}{EI}dx$ is the area of the shaded strip (Fig. 13.2 *b*) within the M/EI diagram.

Since the angle $\left(\theta_B^A\right)$ between the two tangents at *A* and *B* consists of such small angles of elemental sections, we have :

$$\theta_B^A = \sum_B^A d\theta = -\int_A^B \frac{M\,dx}{EI} \quad \text{...(13}$$

In ... $\frac{M}{EI}dx$ is the area of the $\frac{M}{EI}$ diagram between *m* and *n*. T...

...(13.4)

In the above expression, $\frac{M}{EI}dx$ is the area of the $\frac{M}{EI}$ diagram between m and n. The term $\int_B^A \frac{M}{EI}dx$ is therefore the total area of $\frac{M}{EI}$ diagram between B and A, say $\Sigma A_{M/EI}$.

Hence $$\theta_B^A = -\int_B^A \frac{M\,dx}{EI} = -\sum_B^A A_{M/EI} \qquad \text{...(13.5)}$$

If, however EI is constant along the length of the beam, we have

$$EI\,\theta_B^A = -\sum_B^A A_M \qquad \text{...(13.5 a)}$$

where A_M is the area of the B.M. diagram between B and A.

The use of bending moment diagram for calculating the slope and deflection of beam was developed by Mohr. Hence Eq. 13.5 can be stated by the following *first theorem* by Mohr:

Mohr's First Theorem : *The angle θ_B^A between the tangents to the deflection curve at two points A and B is* ***equal*** *to the negative of the area of M/EI diagram between the points.*

(b) Deviation : Let us now determine the deviation d_B^A of B from the tangent at A. Multiplying Eq. 13.3 by x', we have

$$x'\,d\theta = -\frac{M\,.\,x'}{E\,I}\,dx$$

Since the deviation d_B^A consists of summation of each $x'\,.\,d\theta$ of elementary sections, we have

$$d_B^A = \sum_B^A x'\,d\theta = -\int_B^A \frac{M\,x'}{E\,I}\,dx \quad ...(13.5)$$

In the above expression, $\frac{M}{E\,I}\,dx\,.\,x'$ is the moment of the elementary area of the $\frac{M}{E\,I}$ diagram about the point B.

Hence

$$d_B^A = -\sum_B^A A_{M/EI}\,.\,\bar{x} \quad \text{(where } x' = 0 \text{ at } B) \quad ...(13.7)$$

It must be remembered that the variable x' has its origin at the point at which deviation is being computed. Eq. 13.7 can be stated by Mohr's second theorem :

Mohr's Second Theorem : *The deviation of B from tangent at A is equal to the negative of the statical moment (or the first moment) with respect to B, of the $\frac{M}{E\,I}$ diagram area between A and B.*

If however, $E\,I$ is constant along the length of the beam, we have

$$E\,I\,d_B^A = -\sum_B^A A_M\,\bar{x} \quad ...(13.8)$$

where A_M is the area of B.M. diagram between B and A.

Mohr's second theorem is useful is finding the deflection because it relates the position of a point of the beam to the tangent at some other point.

13.4. SIGN CONVENTIONS

The following sign conventions are followed in the area-moment method :

1. B.M. is taken *positive* when it causes *concavity* at the upper side (i.e. if it causes compression in the upper fibres).
2. Area of $M/E\,I$ diagram is taken *positive* if M is *positive.*
3. The rotation θ at any point is taken *positive* if the rotation is *clockwise.*
4. The deflection y at any point is *positive* if it is *downwards* with respect to the original axis of the beam.
5. The angle θ_B^A is *positive* if the tangent at B rotates *clockwise* with respect to tangent at A. Thus θ_B^A in Fig. 13.2(a) is negative since the rotation of tangent at B is anticlockwise with respect to tangent AB' at A.
6. The deviation d_B^A is taken *positive* when it is measured *downward* from the tangent at A. Thus, is Fig 13.2(a), d_B^A is *negative* since it is measured *upward* from the tangent at A.
7. $\bar{x}$ is taken positive when it is measured from point B towards point A.

In the above sign conventions, point B must be to the right of A.

If the area of B.M. diagram is *positive*, the first moment is also positive; hence the *deviation* comes out to be negative from Eq. 13.6 and the point B is above the tangent at A. This situation is illustrated in Fig. 13.2. If, however, as we move from A to B in the direction, the area of B.M. diagram is negative, then the first moment of the area is

Also, $$y_B^A = -\sum_B^A A_{M/EI}\,.\,\bar{x} = -\left[-\frac{\mu L}{EI}\cdot\frac{L}{2}\right] = \frac{\mu L^2}{2EI} \quad \text{(Ans.)}$$

Example 13.4. *A cantilever AB of length L is subjected to a concentrated load W and a couple μ acting at the free end, as shown in Fig. 13.15. Determine the slope and deflection at the free end.*

Solution :

The component B.M. diagrams are shown in Fig. 13.15(*b*) and (*c*) respectively.

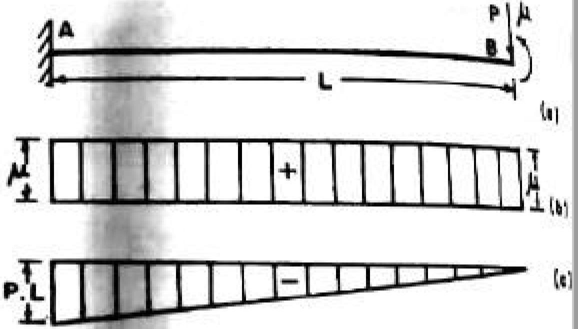


FIG. 13.15

Now $$\theta_B^A = -\sum_B^A A_{M/EI}$$

$$= -\left[\mu L - \frac{1}{2} P L\,.\,L\right]\frac{1}{EI}$$

$$= \frac{1}{2EI}\left[P L - 2\mu\right]$$

and $$y_B^A = -\sum_B^A A_{M/EI}\,.\,\bar{x}$$

$$= -\frac{1}{EI}\left[\mu L\,.\,\frac{L}{2} - \frac{1}{2}\,.\,P L\,.\,L\,.\,\frac{2L}{3}\right]$$

or $$y_B^A = \frac{P L^3}{3EI} - \frac{\mu L^2}{2EI} = \frac{L^2}{6EI}\left[2 P L - 3\mu\right]$$

(2×10^5) ...

Example 12.12. *A beam AB of 6 m span is simply supported at the ends and is loaded as shown in Fig. 12.18. Determine (i) deflection at C (ii) maximum deflection and (iii) slope at end A. Take $E = 2 \times 10^5\ N/mm^2$ and $I = 2000\ cm^4$.*

Solution :

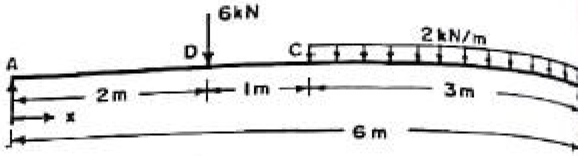


FIG. 12.18

Taking moments about B,

$$R_A = \frac{1}{6}(6 \times 4 + 2 \times 3 \times 1.5) = 5.5 \text{ kN}$$

Measuring x from A, we have

$$EI\frac{d^2y}{dx^2} = -5.5x \Big| + 6(x-2) \Big| + \frac{2(x-3)^2}{2}$$

$$\therefore \quad EI\frac{dy}{dx} = -\frac{5.5x^2}{2} + C_1 \Big| + \frac{6(x-2)^2}{2} \Big| + \frac{2(x-3)^3}{6} \qquad \text{...(i)}$$

and

$$EIy = -\frac{5.5x^3}{6} + C_1x + C_2 \Big| + \frac{6(x-2)^3}{6} \Big| + \frac{2(x-3)^4}{24} \qquad \text{...(ii)}$$

At $x = 0$, $\quad y = 0$. Hence $C_2 = 0$

At $x = 6$ m, $\quad y = 0 = -\frac{5.5}{6}(6)^3 + C_1(6) + \frac{6(6-2)^3}{6} + \frac{2}{24}(6-3)^4$

which gives $\quad C_1 = 21.21$

Hence the slope and deflection equations are :

$$EI\frac{dy}{dx} = -2.75x^2 + 21.21 \Big| + 3(x-2)^2 \Big| + \frac{2}{6}(x-3)^3$$

$$E I y = -\frac{5.5}{6}x^3 + 21.21\,x \Big| + (x-2)^3 \Big| + \frac{2}{24}(x-3)^4 \qquad \text{...(I)}$$

For maximum deflection, $\frac{dy}{dx} = 0$. However, the maximum deflection will be very near mid-point C, say in the sector DC. Hence from (2), including the terms upto point get

$$0 = -2.75\,x^2 + 21.21 + 3\,(x-2)^2$$

$x^2 + 24x - 82.16 = 0$ from which $x = 2.9486$ m.

From II. $E I y_{max} = -\frac{5.5}{6}(2.9486)^3 + 21.21\,(2.9486) + (2.9486 - 2)^3 = 39.89$ kN-m³ units

$$E I y_{max} = 39.89\,(1000)\,(1000)^3 \text{ N-mm}^3 = 39.89 \times 10^{12} \text{ N-mm}^3$$

$$y_{max} = \frac{39.89 \times 10^{12}}{2 \times 10^5 \times 2000\,(10^4)} = \mathbf{9.973\ mm}$$

For deflection at C, put $x = 3$ m in Eq. II, in terms upto point C.

$$E I y_c = -\frac{5.5}{6}(3)^3 + 21.21\,(3) + (3-2)^3 = 39.88 \text{ kN-m}^3 \text{ units}$$

$$y_c = \frac{39.88 \times 10^{12}}{2 \times 10^5\,(2000)\,(10^4)} = \mathbf{9.97\ mm}$$

Thus, we note that the deflection at the mid-span is very nearly equal to the maximum deflection.

Also, putting, $x = 0$ in Eq. (I), we get

$$E I i_A = 21.21 \text{ kN-m}^2 = 21.21 \times 1000\,(1000)^2 \text{ N-mm}^2$$

$$i_A = \frac{21.21 \times 10^9}{2 \times 10^5 \times 2000\,(10^4)} = \mathbf{5.3 \times 10^{-3}\ radian}$$

... with a couple μ at the right hand ... the maximum