Govt. Engineering College Ajmer

III Sem. IT MAIN (2017 – 18)

MM: 10 Sub: Advanced Engg. Mathematics (II Mid–Term Test) Time: 1Hr.

Note: Attempt any 4 questions. Each question carries equal marks.

1. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ using (i) Trapezoidal rule, (ii) Simpson's 1/3 rule and (iii) Simpson's 3/8 rule. Also, compare the results with true value.

2. Using K-T conditions to solve: Min. $f(x,y,z) = x^2 + y^2 + z^2 + 20x + 10y$

   $s.t.\ x \geq 40,\ \ x + y \geq 80$ and $x + y + z \geq 120$

3. Use simplex method to solve: Max. $z = 6x_1 + 4x_2$

   $s.t.\ \ x_1 + x_2 \leq 5,\ x_2 \geq 8$ and $x_1, x_2 \geq 0$

4. Use Runge- Kutta method of fourth order to solve:

   $\frac{dy}{dx} = -2xy^2$; $y(0) = 1$ to find the approximate value of y for x=0.2, 0.4

5. Solve the following difference equation:

   $8y_{n+2} - 6y_{n+1} + y_n = 5\sin\left(\frac{n\pi}{2}\right)$

Hint and Solution:

①

x:	0	1	2	3	4	5	6
$\frac{1}{1+x^2}=y$:	1 (1.0)	1/2 (0.5)	1/5 (0.2)	1/10 (0.1)	1/17 (0.059)	1/26 (0.038)	1/37 (0.027)

(i) $\int_0^6 \frac{dx}{1+x^2} = \frac{1}{2}\left[\left(1+\frac{1}{37}\right)+2\left(\frac{1}{2}+\frac{1}{5}+\frac{1}{10}+\frac{1}{17}+\frac{1}{26}\right)\right] \doteq \frac{1}{2}\left[(1+0.027)+2(0.5+0.2+0.1+0.059+0.038)\right]$

$=1.4105$

(ii) $\int_0^6 \frac{dx}{1+x^2} = \frac{1}{3}\left[(1+0.027)+4(0.5+0.1+0.038)+2(0.2+0.059)\right] = 1.3657$

(iii) $\int_0^6 \frac{dx}{1+x^2} = \frac{3}{8}\left[(1+0.027)+3(0.5+0.2+0.059+0.038)+2(0.1)\right] = 1.3568$

& true value $= \int_0^6 \frac{dx}{1+x^2} = \tan^{-1}6 - \tan^{-1}0 = 1.4056 - 0 = 1.4056$

② $\frac{\partial L}{\partial x} = 0 \Rightarrow 2x+20+\lambda_1+\lambda_2+\lambda_3=0$, $\frac{\partial L}{\partial y}=0 \Rightarrow 2y+10+\lambda_2+\lambda_3=0$

& $\frac{\partial L}{\partial z} = 0 \Rightarrow 2z+\lambda_3=0$

Also, $\lambda_1 g_1 = 0 \Rightarrow \lambda_1(x-40)=0$, $\lambda_2 g_2 = 0 \Rightarrow \lambda_2(x+y-80)=0$

& $\lambda_3 g_3 = 0 \Rightarrow \lambda_3(x+y+z-120)=0$

as well as $x\geq 40$, $x+y\geq 80$, $x+y+z\geq 120$ & $\lambda_1, \lambda_2, \lambda_3 \leq 0$

Let $\lambda_1, \lambda_2, \lambda_3 \neq 0 \Rightarrow x=40, y=40, z=40$, which satisfies all K-T conditions

So, min $f = 6000$

③ $z_j - c_j$ for table-1: $-6, -m-4, 0, m, 0$

& for table-2, all $z_j - c_j \geq 0$ with $A_1 = 3$, $x_2 = 5$ in the soln.

⇓ NOT feasible sol.

④ For $y(x=0.2)$, $k_1=0$, $k_2=-0.04$, $k_3=-0.0384$, $k_4=-0.074 \Rightarrow k=-0.0385$

$\Rightarrow y(x=0.2) = 0.9615$

& for $y(x=0.4)$, $k=-0.074$, $k_2=-0.1026$, $k_3=-0.0994$, $k_4=-0.1182$

$\Rightarrow k=-0.0995 \Rightarrow y(0.4)=0.862$

⑤ $(8E^2-6E+1)\,y_n = 5\cdot\sin\frac{n\pi}{2}$

A.E. is $8m^2-6m+1=0 \Rightarrow m=\frac{1}{4}, \frac{1}{2}$

$\Rightarrow$ C.F. $= c_1\left(\frac{1}{4}\right)^n + c_2\left(\frac{1}{2}\right)^n$

& P.I. $= \frac{1}{(4E-1)(2E-1)}\, 5\sin\frac{n\pi}{2} = 5\times \text{I.P. of } \frac{1}{(4E-1)(2E-1)} e^{in\pi/2}$

$= \frac{1}{17}\left(6\cos\frac{n\pi}{2} - 7\sin\frac{n\pi}{2}\right)$

Thus, reqd. Soln is $y_n = c_1\left(\frac{1}{4}\right)^n + c_2\left(\frac{1}{2}\right)^n + \frac{1}{17}\left(6\cos\frac{n\pi}{2} - 7\sin\frac{n\pi}{2}\right)$