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GOVT ENGINEERING COLLEGE AJMER,

Dept of ECE,

I Midterm Test 2017-18,

VIII Sem ECE

Max Marks: 10 Subject: Radar & TV Engineering Time: 1 HR.

Q1. Derive the expression for Radar Range Equation having Noise Figure as one of the parameter. [3]

Q2. Derive the mathematical expression for the Blind Speed using frequency response of the delay line canceller in a MTI radar. [4]

Q3. What is Doppler effect and explain how Doppler frequency is related with the speed/velocity of the target. [3]

Detailed Solution to Question No. 1.

Deriving the Radar Range Equation

The essence of radar is the ability to scan three-dimensional space and gather information about detected objects, ranging from simple presence to details such as location, speed, direction, shape, and identity. In most implementations, a pulsed-RF or pulsed-microwave signal is generated by the radar system, beamed toward the target in question, and collected by the same antenna that transmitted the signal.

The signal power at the radar receiver is directly proportional to the transmitted power, the antenna gain (or aperture size), and the degree to which a target reflects the radar signal (i.e., its RCS). Perhaps more significantly, it is indirectly proportional to the fourth power of the distance to the target.

This entire process is described by the radar range equation. It incorporates the crucial variables and provides a basis for understanding the measurements that are made to verify and ensure optimal performance.

Our derivation of the range equation starts with a simple spherical scattering model of propagation for a point-source antenna (i.e., an isotropic radiator). Assume, for simplicity, that the antenna is illuminating the interior of an imaginary sphere with equal power density in each unit of surface area (Figure 1). The surface area of a sphere is a function of its radius:

$$A_s = 4\pi R^2$$

A_s = area of a sphere
R = radius of the sphere

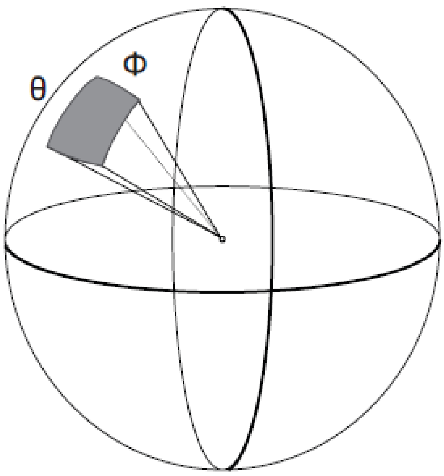

Figure 1. Ideal isotropic antenna radiation produces equal power density in each unit of surface area.

The power density is found by dividing the total transmit power, in watts, by the surface area of the sphere in square meters:

$$r = \frac{P_t}{A_s} = \frac{P_t}{4\pi R^2}$$

r = power density in watts per square meters
P_t = total transmitted power in watts

Because radar systems use directive antennas to focus radiated energy onto a target, the equation can be modified to account for the directive gain G of the antenna. This is defined as the ratio of power directed toward the target compared to the power from an ideal isotropic antenna:

$$r_T = \frac{P_t G_t}{4\pi R^2}$$

r_T = power density directed toward the target from the directive antenna
G_t = gain of the directive antenna

This equation describes the transmitted power density that strikes the target. Some of that energy will be reflected in various directions and some will be reradiated back to the radar system. The amount of incident power density that is reradiated back to the radar is a function of the RCS or s of the target. RCS has units of area and is a measure of target size, as seen by the radar (Figure 2).

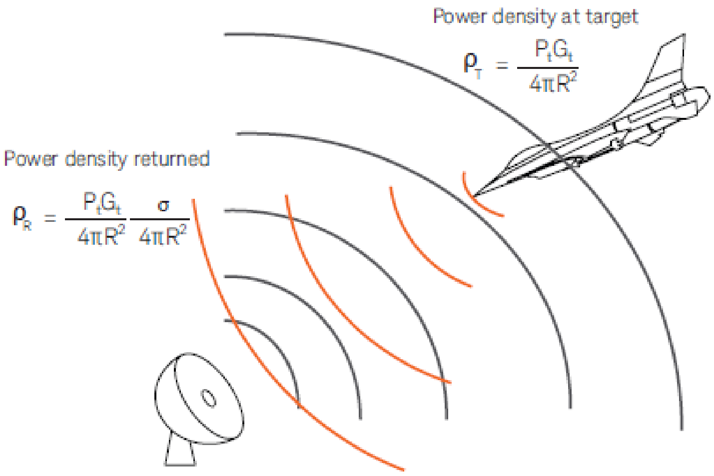

Figure 2. The reflected power density returned to the radar is proportional to the power density of the transmitted signal at the target, and it also affected by the RCS of the target.

With this information, the equation can be expanded to solve for the power density returned to the radar antenna. This is done by multiplying the transmitted power density by the ratio of the RCS and area of the sphere:

$$r_R = \frac{P_t G_t}{4\pi R^2} \frac{s}{4\pi R^2}$$

r_R = power density returned to the radar, in watts per square meter
s = RCS in square meters

Thus, the radar antenna will receive a portion of this signal reflected by the target. This signal power is equal to the return power density at the antenna multiplied by the effective area, A_e of the antenna:

$$S = \frac{P_t G_t s A_e}{4\pi R^4}$$

S = signal power received at the receiver in watts
P_t = transmitted power in watts
G_t = gain of transmit antenna (ratio)
s = RCS in square meters
R = radius or distance to the target in meters
A_e = effective area of the receive antenna square meters

Antenna theory allows us to relate the gain of an antenna to its effective area as follows:

$$A_e = \frac{G_r l^2}{4\pi}$$

G_r = gain of the receive antenna
l = wavelength of the radar signal in meters

The equation for the received signal power can now be simplified. Note that for a monostatic radar the antenna gain G_t and G_r are equivalent. This is assumed to be the case for this derivation:

$$S = \frac{P_t G_t G_r \; l^2 s}{(4\pi)^2 R^4 \, 4\pi}$$

$$\longrightarrow \quad S = \frac{P_t G^2 \; l^2 s}{(4\pi)^3 R^4}$$

S = signal power received at the receiver in watts
P_t = transmitted power in watts
G = antenna gain (assume same antenna for transmit and receive)
l = wavelength of the radar signal in meters
s = RCS of the target in square meters
R = radius or distance to the target in meters

Now that the signal power at the receiver is known, the next step is to analyze how the receiver will process the signal and extract information. The primary factor limiting the receiver is noise and the resulting signal-to-noise (S/N) ratio.

The theoretical limit of the noise power at the input of the receiver is described as Johnson noise or thermal noise. It is a result of the random motion of electrons and is proportional to temperature:

$$N = kTB_n$$

N = noise power in watts
k = Boltzmann's constant (1.38×10^{-23} J/K)
T = temperature in Kelvin
B_n = system noise bandwidth

At a room temperature of 290 K, the available noise power at the input of the receiver is 4×10^{-21} W/Hz, –203.98 dBW/Hz, or –173.98 dBm/Hz. The available noise power at the output of the receiver will always be higher than predicted by the above equation due to noise generated within the receiver.[1] From this, the output noise will be equal to the ideal noise power multiplied by the noise factor and gain of the receiver:

$$N_o = GF_n kTB$$

N_o = total receiver noise
G = gain of the receiver
F_n = noise factor

The gain of the receiver can be rewritten as the ratio of the signal output of the receiver to the signal input ($G = S_o/S_i$). Solving for the noise factor F_n yields the following equation:

$$F_n = \frac{S_i / N_i}{S_o / N_o} \quad \text{Where } N_i = kTB$$

By definition, the noise factor is the ratio of the S/N in to the S/N out. The equation can then be rewritten in a different form, and again $G = S_o/S_i$:

$$F_n = \frac{N_o}{kT_oB_nG}$$

N_o = total receiver noise
G = gain of the receiver
S_o = receiver output signal
S_i = receiver input signal
T_o = room temperature
k = Boltzmann's constant
B_n = receiver noise bandwidth

Because noise factor describes the degradation of signal-to-noise as the signal passes through the system, the minimum detectable signal (MDS) at the input can be determined. It corresponds to a minimum output S/N ratio with an input noise power of kTB, and S_i approaches S_{min} when the minimum S_o/N_o condition is met:

$$S_{min} = kT_oB_nF_n\left(\frac{S_o}{N_o}\right)_{min}$$

S_{min} = minimum power required at input of the receiver
F_n = noise factor
$(S_o/N_o)_{min}$ = minimum ratio required by the receiver processor to detect the signal

Now that the minimum signal level required to overcome system noise is defined, the maximum range of the radar can be calculated by equating the MDS (S_{min}) to the signal level reflected from the target at maximum range. Setting S_{min} equal to the earlier equation for S yields the following:

$$S_{min} = kT_oB_nF_n\left(\frac{S_o}{N_o}\right)_{min} = \frac{P_tG^2l^2s}{(4\pi)^3R^4_{max}}$$

Rearranging this equation, we can solve for the maximum range of the radar:

$$R^4_{max} = \frac{P_tG^2l^2s}{kT_oB_nF_n(S/N)(4\pi)^3}$$

P_t = transmitted power in watts
G = antenna gain (assume same antenna for transmit and receive)
l = wavelength of radar signal in meters
s = RCS of target in square meters
k = Boltzmann's constant
T = room temperature in Kelvin
B_n = receiver noise bandwidth in hertz
F_n = noise factor
S/N = minimum signal-to-noise ratio required by receiver processor to detect the signal

The equation now describes the maximum target range of the radar as a function of transmitter power, antenna gain, target RCS, system noise figure, and minimum S/N ratio. In reality, this is a simplistic model of system performance. Many other factors also affect performance, and this includes modifications to the assumptions made to derive this equation.

Detailed Solution to Q2.

Delay Line Canceller:-

The simple MTI delay-line canceller shown in Fig. 4.6 is an example of a time-domain filter.The capability of this device depends on the quality of the medium used as the delay line. Thedelay line must introduce a time delay equal to the pulse repetition interval. For typical ground-based air-surveillance radars this might be several milliseconds. Delay times of this magnitude cannot be· achieved with practical electromagnetic transmission lines. By converting the electromagnetic signal to an acoustic signal it is possible to utilize delay lines of a delay line must introduce a time delay equal.

One of the advantages of a time-domain delay-line canceler as compared to the more conventional frequency-domain filter is that a single network operates at all ranges and does not require a separate filter for each range resolution cell. Frequency-domain doppler filter banks are of interest in some forms of MTI and pulse-doppler radar.

Frequency Response of Delay Line canceller

The delay-line canceler acts as a filter which rejects the d-c component of clutter. Because of its periodic nature, the filter also rejects energy in the vicinity of the pulse repetition frequency and its harmonics.

The signal from a target at range R_0, the output of the phase detector can be given as:

$$V_1 = k\sin(2\pi f_d t - \phi_0) \qquad \ldots (5)$$

Where f_d is Doppler frequency, ϕ_0 constant phase of $4\pi R_0/\lambda$. The signal from the previous radar transmission is similar, which is delayed by time T_P

$$V_2 = k\sin[2\pi f_d(t - T_P) - \phi_0] \qquad \ldots (6)$$

Everything else is assumed to remain essentially constant over the interval T_P so that k is the same for both pulses. The output from the subtractor is

$$V = V_1 - V_2 = 2k\sin(\pi f_d T_P)\cos\left[2\pi f_d(t - \frac{T_P}{2}) - \phi_0\right] \qquad \ldots (7)$$

The magnitude of the relative frequency-response of the delay-line canceler [ratio of the amplitude of the output from the delay-line canceler, $2k\sin(\pi f_d T_P)$, to the amplitude of the normal radar video k] is shown in Fig. 4.8.

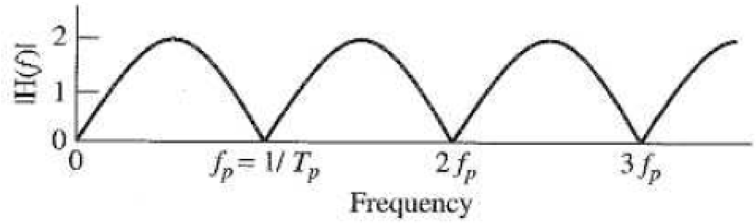

Fig. 4.8 Frequency response of the single delay-line canceller; T = delay time $=1/f_p$

Blind Speed:-

The response of the single-delay-line canceler will be zero whenever the argument $\pi f_d T_P$ in the amplitude factor of Eq. (7) is 0, π, 2 π, ... , etc., or when

$$f_d = \frac{2V_r}{\lambda} = \frac{n}{T_P} = nf_p \quad n = 0,1,2,3,\ldots\ldots \qquad \ldots (8)$$

The delay-line cancela not only eliminates the d-c component caused by clutter (n = 0), but unfortunately it also rejects any moving target whose doppler frequency happens to be the same as the prf or a multiple

there of. Those relative target velocities which result in zero MTI response are called blind speed and can be given as:

$$v_n = \frac{n\lambda}{2T_p} = \frac{n\lambda f_p}{2} \quad n = 0,1,2,3,\ldots\ldots \qquad \ldots (9)$$

where v_n is the nth blind speed. If λ is measured in meters,fp in Hz, and the relative velocity in knots, the blind speeds are

$$v_n = \frac{n\lambda f_p}{1.02} \approx n\lambda f_p \qquad \dots (10)$$

The blind speeds are one of the limitations of pulse MTI radar which do not occur with CW radar. They are present in pulse radar because doppler is measured by discrete samples (pulses) at the prf rather than continuously.

Solution to Q3.

It is well known in the fields of optics and acoustics that if either the source of oscillation or the observer of the oscillation is in motion, an apparent shift in frequency will result. This is the doppler effect and is the basis of CW radar.

If R is the distance from the radar to target, the total number of wavelengths (λ) contained in the two-way path between the radar and the target is 2R/λ. The distance R and the wavelength (λ), are assumed to be measured in the same units. Since one wavelength corresponds to an angular excursion of 2π radians, the total angular excursion ϕ made by the electromagnetic wave during its transit to and from the target is $4\pi R/\lambda$.

If target is in motion the range R and phase φ is continually changing. Thus the change in phase with respect to time can be given as frequency.

$$\frac{d\phi}{dt} = \frac{4\pi}{\lambda}\frac{dR}{dt} \qquad \dots (1)$$

Range with respect to time can be defined as the radial velocity of the target. Thus the Doppler angular frequency can be given as:

$$\omega_d = 2\pi f_d = \frac{4\pi}{\lambda} v_r \qquad \dots (2)$$

Where f_d is Doppler frequency and v_r is the radial velocity of the target with respect to radar. The Doppler frequency can be related with transmitter frequency f_0.

$$f_d = \frac{2v_r}{\lambda} = \frac{2v_r f_0}{c} \qquad \dots (3)$$

When v_r is given in knots then the Doppler frequency can be given as:

$$f_d = \frac{1.03 v_r\,(knots)}{\lambda(m)} \qquad \dots (4)$$

The relative velocity may be written $v_r = v\cos\theta$ where v is the target speed and θis the Angle made by the target trajectory and the line joining radar and target. When θ = 0, the doppler frequency is maximum. The doppler is zero when the trajectory is perpendicular to the radar line of sight (θ= 90°).