DEMAND FORECASTING

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Learning Objectives

- Understand techniques to foresee the future

What is Forecasting?

- Process of predicting a future event
- Underlying basis of all business decisions
  - Production
  - Inventory
  - Personnel
  - Facilities

Types of Forecasts by Time Horizon

- Short-range forecast
  - Up to 1 year; usually < 3 months
  - Job scheduling, worker assignments, work force level
- Medium-range forecast
  - 3 months to 3 years
  - Sales & production planning, budgeting, Inventory
- Long-range forecast
  - 3+ years
  - Types of products and services to offer
  - Facility and equipment levels
  - Facility location

Influence of Product Life Cycle

- Stages of introduction & growth require longer-range forecasts than maturity and decline
- Forecasts useful in projecting
  - staffing levels,
  - inventory levels, and
  - factory capacity

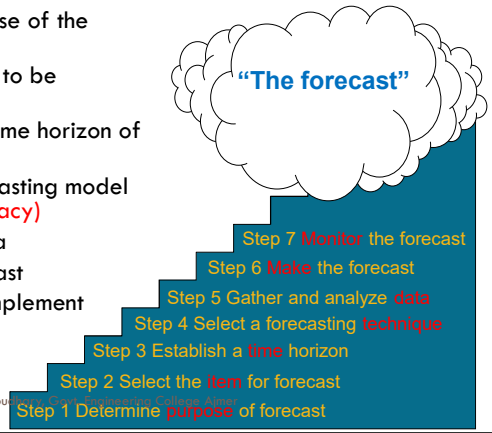
  as product passes through stages

Demand forecasts

- Predict *new* product sales
- Predict *existing* product sales

Seven Steps in Forecasting

- Determine the use of the forecast
- Select the items to be forecast
- Determine the time horizon of the forecast
- Select the forecasting model (Cost and accuracy)
- Gather the data
- Make the forecast
- Validate and implement results



Realities of Forecasting

- Forecasts are seldom perfect because of randomness
- Short-term forecasts tend to be more accurate than longer-term forecasts
  - Forecast accuracy decreases as time horizon increases
- Both product family and aggregated product forecasts are more accurate than individual product forecasts
  - Forecasts more accurate for groups vs. individuals

Forecasting Approaches

Qualitative Methods

- Used when situation is vague & little data exist
  - New products
  - New technology
- Involves intuition, experience
- e.g., forecasting sales on Internet

Quantitative Methods

- Used when situation is 'stable' & historical data exist
  - Existing products
  - Current technology
- Involves mathematical techniques
- e.g., forecasting sales of color televisions

Overview of Qualitative Methods

- Jury of executive opinion
  - Pool opinions of high-level executives, sometimes augment by statistical models
- Sales force composite
  - estimates from individual salespersons are reviewed for reasonableness, then aggregated
- Delphi method
  - Panel of experts, queried iteratively
- Consumer Market Survey
  - Ask the customer

Jury of Executive Opinion

- Involves small group of high-level managers
- Group estimates demand by working together
- Combines managerial experience with statistical models
- Relatively quick
- 'Group-think' disadvantage

Sales Force Composite

- Each salesperson projects their sales
- Combined at district & national levels
- Sales rep's know customers' wants
- Tends to be overly optimistic

Delphi Method

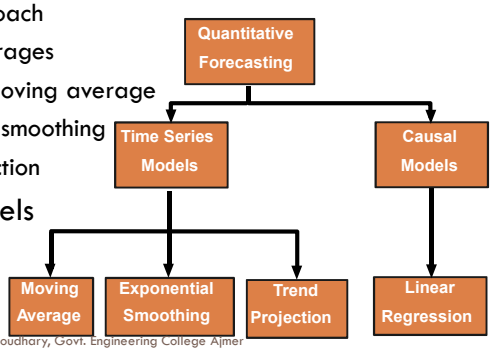
- Iterative group process
- 3 types of people
  - Decision makers
  - Staff
  - Respondents
- Reduces 'group-think'

Consumer Market Survey

- Ask customers about purchasing plans
- What consumers say, and what they actually do are often different
- Sometimes difficult to answer

Overview of Quantitative Approaches

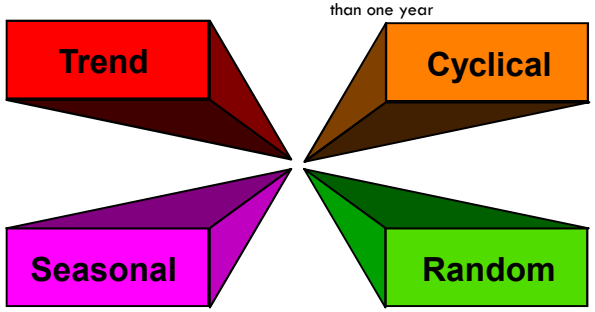
- Time-series models
  - Naïve approach
  - Moving averages
  - Weighted moving average
  - Exponential smoothing
  - Trend projection
- Causal models



Time series models

Trend - long-term movement in data

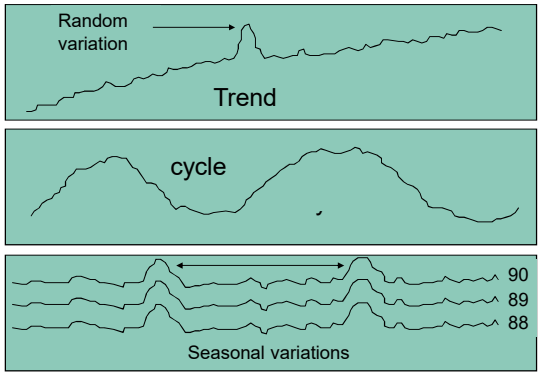
CYCLE- wave like variations lasting more than one year



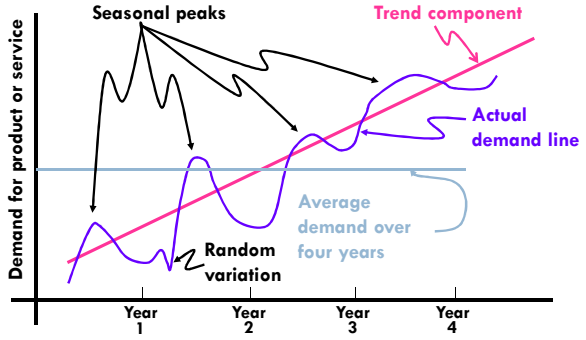
Seasonality - short-term regular variations in data

Random variations - caused by chance or unusual circumstances

Time series models

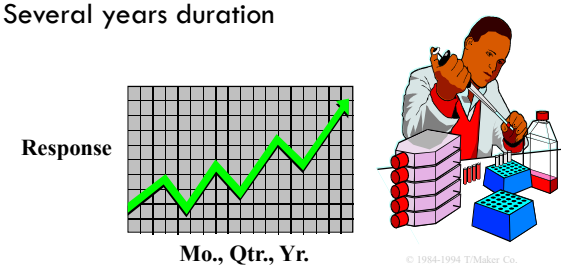


Time series models



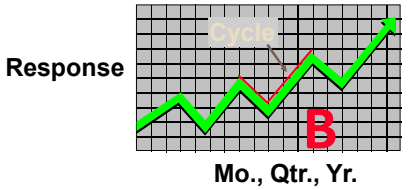
Trend Component

- Persistent, overall upward or downward pattern
- Due to population, technology etc.
- Several years duration



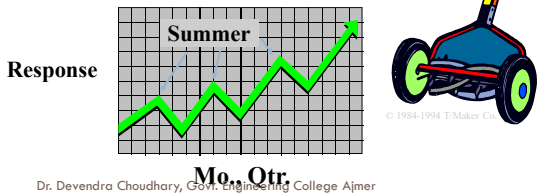
Cyclical Component

- Repeating up & down movements
- Due to interactions of factors influencing economy
- Usually 2-10 years duration



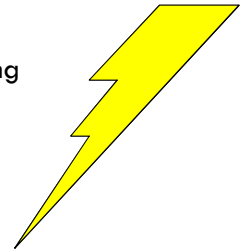
Seasonal Component

- Regular pattern of up & down fluctuations
- Due to weather, customs etc.
- Occurs within 1 year



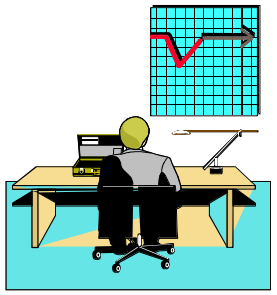
Random Component

- Erratic, unsystematic, 'residual' fluctuations
- Due to random variation or unforeseen events
  - Union strike
  - Tornado
- Short duration & nonrepeating



Naive Approach

- Assumes demand in *next* period is the same as demand in *most recent* period
- e.g., If May sales were 48, then June sales will be 48
- Cannot provide high accuracy
- Sometimes cost effective & efficient



Moving Average Method

- MA is a series of arithmetic means
- Used if little or no trend
- Used often for smoothing
  - Provides overall impression of data over time
- Increasing *n* makes forecast less sensitive to changes
- Do not forecast trend well
- Require much historical data

$$MA = \frac{\sum \text{Demand in Previous } n \text{ Periods}}{n}$$

Weighted Moving Average Method

More recent values in a series are given more weight in computing the forecast.

$$WMA = \frac{\sum \text{Weight} * \text{Demand}}{\text{Sum of weights}}$$

Exponential Smoothing

- We should give more weight to the more recent time periods when forecasting.
- Need just three pieces of data to start:
  - Last period's forecast (F_t)
  - Last periods actual demand value (A_t)
  - Select value of **smoothing coefficient, α** ,between 0 and 1.0
- If no last period forecast is available, average the last few periods or use naive method

$$F_{t+1} = \alpha A_t + (1-\alpha)F_t$$

$$F_{t+1} = F_t + \alpha(A_t - F_t)$$

Forecast Effect of Smoothing Constant α

- Higher α values (e.g. 0.7 or 0.8) may place too much weight on last period's random variation

$$F_t = \alpha A_{t-1} + \alpha(1-\alpha) A_{t-2} + \alpha(1-\alpha)^2 A_{t-3} + \ldots$$

α=	Weights		
	Prior Period	2 periods ago	3 periods ago
	α	α(1 - α)	α(1 - α)²
α= 0.10	10%	9%	8.1%
α= 0.90	90%	9%	0.9%

Problem 1

- **Determine forecast for periods 7 & 8**
- **3-period** moving average
- **5-period** moving average
- **3-period** weighted moving average with t-1 weighted 0.5, t-2 weighted 0.3 and t-3 weighted 0.2
- **Exponential smoothing** with alpha=0.2 and 0.6, the period 6 forecast being 375

Period	Actual
1	300
2	315
3	290
4	346
5	320
6	360
7	376
8	

Problem 1

Period	Actual	3-P MA	5-P MA	3-P WMA	ES (α = .2)	ES (α = .6)
1	300					
2	315					
3	290					
4	346					
5	320					
6	360					
7	376	342				
8		352				

Problem 1

Period	Actual	3-P MA	5-P MA	3-P WMA	ES (α = .2)	ES (α = .2)
1	300					
2	315					
3	290					
4	346					
5	320					
6	360					
7	376		326.2			
8			338.4			

Problem 1

Period	Actual	3-P MA	5-P MA	3-P WMA	ES (α = .2)	ES (α = .2)
1	300					
2	315					
3	290					
4	346					
5	320					
6	360					
7	376			345.2		
8				360		

0.5*360+0.3*320+0.2*346

Problem 1

Period	Actual	3-P MA	5-P MA	3-P WMA	ES (α = .2)	ES (α = .2)
1	300					
2	315					
3	290					
4	346					
5	320					
6	360				375	
7	376				372	
8					372.8	

Given

$F_7 = F_6 + \alpha^*(A_6 - F_6)$

$F_8 = F_7 + \alpha^*(A_7 - F_7)$

Problem 1

Period	Actual	3-P MA	5-P MA	3-P WMA	ES (α = .2)	ES (α = .2)
1	300					
2	315					
3	290					
4	346					
5	320					
6	360					375
7	376					366
8						372

Given

$F_7 = F_6 + \alpha^*(A_6 - F_6)$

$F_8 = F_7 + \alpha^*(A_7 - F_7)$

Problem 1

Period	Actual	3-P MA	5-P MA	3-P WMA	ES (α = .2)	ES (α = .2)
1	300					
2	315					
3	290					
4	346					
5	320					
6	360					
7	376	342	326.2	345.2	372	366
8		352	338.4	360	372.8	372

Forecast Accuracy

- Error - difference between actual value and predicted value $E_t = A_t - F_t$
- Mean Absolute Deviation (MAD)
  - Average absolute error $MAD = \frac{\sum |\text{actual} - \text{forecast}|}{n}$
- Mean Squared Error (MSE)
  - Average of squared error $MSE = \frac{\sum (\text{actual} - \text{forecast})^2}{n}$
- Mean Absolute Percent Error (MAPE)
  - Average absolute percent error $MAPE = \frac{\sum |A_t - F_t|}{\sum A_t}$

Which α is better

Period	Actual	Forecast α=.10	Forecast α=.50
1	180	175	175
2	168		
3	159		
4	175		
5	190		
6	205		
7	180		
8	182		
9	?	?	?

Given

To determine

Which α is better

Period	Actual	Forecast α=.10	Forecast α=.50
1	180	175.00	175.00
2	168	175.50	177.50
3	159	174.75	172.75
4	175	173.18	165.88
5	190	173.36	170.44
6	205	175.02	180.22
7	180	178.02	192.61
8	182	178.22	186.30
9	?	178.60	184.15

Which α is better

Period	Actual	Forecast α=.10	Error	Absolute Error	Squared error	Absolute % error
1	180	175.00	5.00	5.00	25.00	2.78
2	168	175.50	-7.50	7.50	56.25	4.46
3	159	174.75	-15.75	15.75	248.06	9.91
4	175	173.18	1.82	1.82	3.33	1.04
5	190	173.36	16.64	16.64	276.97	8.76
6	205	175.02	29.98	29.98	898.70	14.62
7	180	178.02	1.98	1.98	3.92	1.10
8	182	178.22	3.78	3.78	14.31	2.08
			4.49	10.31	190.82	5.59
			ME	MAD	MSE	MAPE

Which α is better

Period	Actual	Forecast α=.50	Error	Absolute Error	Squared error	Absolute % error
1	180	175.00	5.00	5.00	25.00	2.78
2	168	177.50	-9.50	9.50	90.25	5.65
3	159	172.75	-13.75	13.75	189.06	8.65
4	175	165.88	9.13	9.13	83.27	5.21
5	190	170.44	19.56	19.56	382.69	10.30
6	205	180.22	24.78	24.78	614.11	12.09
7	180	192.61	-12.61	12.61	159.00	7.01
8	182	186.30	-4.30	4.30	18.53	2.37
			2.29	12.33	195.24	6.76
			ME	MAD	MSE	MAPE

Which α is better

	Forecast α=.10	Forecast α=.50
ME	4.49	2.29
MAD	10.31	12.33
MSE	190.82	195.24
MAPE	5.59	6.76

On the basis of MAD, MSE and MAPE, a smoothing constant of α = 0.10 is Preferred to α = 0.50

Trend-adjusted exponential smoothing

Month	Actual	Forecast $\alpha = 0.4$	Forecast $\alpha = 0.9$
1	100	100 (Given)	100 (Given)
2	200	100	100
3	300	140	190
4	400	204	289
5	500	282.4	388.9

- ES fails to respond to trends.
- To improve our forecast, ES is adjusted for trend.
- With T-ES, both the forecast and trend are smoothed using two smoothing constants α and β, respectively.

Trend-adjusted exponential smoothing

- It uses a three step process
  - **Step 1 - Smoothing the forecast**

    $F_t = \alpha$(Actual demand last period) + (1- α)(Forecast last period+Trend estimate last period)

    $F_t = \alpha(A_{t-1}) + (1-\alpha)(F_{t-1} + T_{t-1})$
  - **Step 2 – Smoothing the trend**

    $T_t = \beta$(Forecast this period - Forecast last period) + (1- β)(Trend estimate last period)

    $T_t = \beta(F_t - F_{t-1}) + (1-\beta)T_{t-1}$
  - **Step 3 – Forecast including the trend**

    Forecast including trend (FIT_t) = exponentially smoothed forecast (F_t) + exponentially smoothed trend (T_t)

Forecasting trend problem

- A company uses exponential smoothing with trend to forecast usage of its lawn care products. At the end of July the company wishes to forecast sales for August. July demand was 62. The trend in July has been 15 additional gallons of product sold per month. Forecast has been *57* gallons per month. The company uses alpha+0.2 and beta +0.10. Forecast for August.

- **Smooth the forecast:**

  $F_{August} = \alpha * A_{July} + (1-\alpha)(F_{July} + T_{July}) = 0.2*62 + 0.8*(57+15) = 70$
- **Smooth the trend:**

  $T_{August} = \beta(F_{August} - F_{July}) + (1-\beta)*T_{July} = 0.1*(70-57) + 0.9*15 = 14.8$
- **Forecast including trend:**

  $FIT_{August} = F_{August} + T_{August} = 70+14.8 = 84.8$ gallons

T-ES Problem

- A manufacturer wants to forecast demand for an item. Past sales indicates an increasing trend. The firm assumes the initial forecast of month 1 was 11 units and the trend over that period was 2 units. Take α = .2 and β = .4.

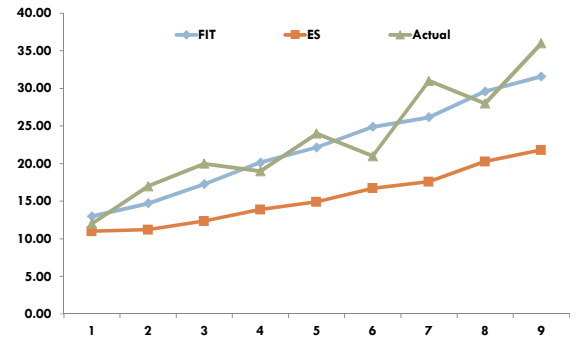
Month	Actual
1	12
2	17
3	20
4	19
5	24
6	21
7	31
8	28
9	36
10	?

T-ES Problem

$F_t = \alpha(A_{t-1}) + (1-\alpha)(F_{t-1} + T_{t-1})$ $\quad$ $T_t = \beta(F_t - F_{t-1}) + (1-\beta)T_{t-1}$

Month	Actual		Forecast		Trend	FIT
1	12		11.00		2.00	13.00
2	17	F2=.2*12+(1-.2)*(11+2)	12.80	T2=.4*(12.8-11)+(1-.4)*2	1.92	14.72
3	20		15.18		2.10	17.28
4	19		17.82		2.32	20.14
5	24		19.91		2.23	22.14
6	21		22.51		2.38	24.89
7	31		24.11		2.07	26.18
8	28		27.14		2.45	29.59
9	36		29.28		2.32	31.60
10	?		32.48		2.68	35.16

Comparing ES and T-ES



Forecasting Seasonality

- Calculate the average demand per season
  - *E.g.*: average monthly/quarterly demand
- Calculate the average demand over the all season
- Calculate a seasonal index for each season :
  - Divide the average monthly/quarterly demand of each season by the average demand over the all season
- Forecast demand for the next year & divide by the number of seasons
  - Use regular forecasting method & divide by four for average quarterly demand
- Multiply next year's average seasonal demand by each seasonal index
  - Result is a forecast of demand for each month/quarter of next year

Forecasting Seasonality

- Seasonality problem: a university must develop forecasts for the next year's quarterly enrollments. It has collected quarterly enrollments for the past two years. It has also forecast total enrollment for next year to be 90,000 students. What is the forecast for each quarter of next year?

Quarter	Year 1	Year 2	Average	Seasonal Index	Year3
Fall	24000	26000	25000	1.22	27439
Winter	23000	22000	22500	1.10	24695
Spring	19000	19000	19000	0.93	20854
Summer	14000	17000	15500	0.76	17012
Total	80000	84000	20500		90000

Controlling the Forecast

- Control chart
  - A visual tool for monitoring forecast errors
  - Used to detect non-randomness in errors
- Forecasting errors are in control if
  - All errors are within the control limits
  - No patterns, such as trends or cycles, are present

Tracking Signal

- Ratio of cumulative error to MAD

$$\text{Tracking signal} = \frac{\sum(\text{Actual - forecast})}{\text{MAD}}$$

- Measures how well forecast is predicting actual values
- Monitors the forecast to see if it is biased high or low
- Should be within upper and lower control limits
- If the forecasting model is performing well, the TS should be around zero

Tracking Signal Computation

Mo	Fcst	Act	Error	RSFE	Abs Error	Cum Error	MAD	TS
1	100	90						
2	100	95						
3	100	115						
4	100	100						
5	100	125						
6	100	140						

Tracking Signal Computation

Mo	Forc	Act	Error	RSFE	Abs Error	Cum Error	MAD	TS
1	100	90	-10					
2	100	95						
3	100	115						
4	100	100						
5	100	125						
6	100	140						

Error = Actual - Forecast
= 90 - 100 = -10

Tracking Signal Computation

Mo	Forc	Act	Error	RSFE	Abs Error	Cum |Error|	MAD	TS
1	100	90	-10	-10				
2	100	95						
3	100	115						
4	100	100						
5	100	125						
6	100	140						

RSFE = Σ Errors
= NA + (-10) = -10

Tracking Signal Computation

Mo	Forc	Act	Error	RSFE	Abs Error	Cum |Error|	MAD	TS
1	100	90	-10	-10	10			
2	100	95						
3	100	115						
4	100	100						
5	100	125						
6	100	140						

Abs Error = |Error|
= |-10| = 10

Tracking Signal Computation

Mo	Forc	Act	Error	RSFE	Abs Error	Cum |Error|	MAD	TS
1	100	90	-10	-10	10	10		
2	100	95						
3	100	115						
4	100	100						
5	100	125						
6	100	140						

Cum |Error| = Σ |Errors|
= NA + 10 = 10

Tracking Signal Computation

Mo	Forc	Act	Error	RSFE	Abs Error	Cum |Error|	MAD	TS
1	100	90	-10	-10	10	10	10.0	
2	100	95						
3	100	115						
4	100	100						
5	100	125						
6	100	140						

MAD = Σ |Errors|/n
= 10/1 = 10

Tracking Signal Computation

Mo	Forc	Act	Error	RSFE	Abs Error	Cum |Error|	MAD	TS
1	100	90	-10	-10	10	10	10.0	-1
2	100	95						
3	100	115						
4	100	100						
5	100	125						
6	100	140						

TS = RSFE/MAD
= -10/10 = -1

Tracking Signal Computation

Mo	Forc	Act	Error	RSFE	Abs Error	Cum |Error|	MAD	TS
1	100	90	-10	-10	10	10	10.0	-1
2	100	95	-5					
3	100	115						
4	100	100						
5	100	125						
6	100	140						

Error = Actual - Forecast
= 95 - 100 = -5

Tracking Signal Computation

Mo	Forc	Act	Error	RSFE	Abs Error	Cum |Error|	MAD	TS
1	100	90	-10	-10	10	10	10.0	-1
2	100	95	-5	-15				
3	100	115						
4	100	100						
5	100	125						
6	100	140						

RSFE = Σ Errors
= (-10) + (-5) = -15

Tracking Signal Computation

Mo	Forc	Act	Error	RSFE	Abs Error	Cum |Error|	MAD	TS
1	100	90	-10	-10	10	10	10.0	-1
2	100	95	-5	-15	5			
3	100	115						
4	100	100						
5	100	125						
6	100	140						

Abs Error = |Error|
= |-5| = 5

Tracking Signal Computation

Mo	Forc	Act	Error	RSFE	Abs Error	Cum |Error|	MAD	TS
1	100	90	-10	-10	10	10	10.0	-1
2	100	95	-5	-15	5	15		
3	100	115						
4	100	100						
5	100	125						
6	100	140						

Cum Error = Σ |Errors|
= 10 + 5 = 15

Tracking Signal Computation

Mo	Forc	Act	Error	RSFE	Abs Error	Cum |Error|	MAD	TS
1	100	90	-10	-10	10	10	10.0	-1
2	100	95	-5	-15	5	15	7.5	
3	100	115						
4	100	100						
5	100	125						
6	100	140						

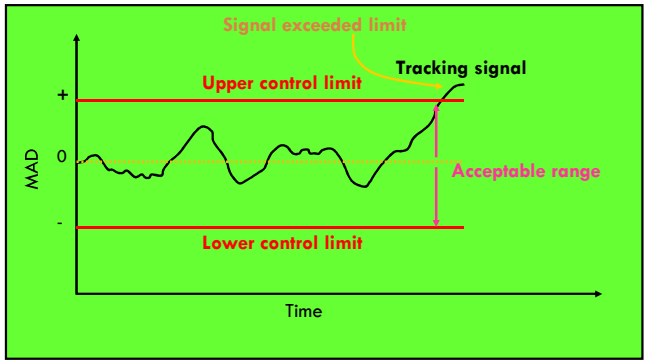
MAD = Σ |Errors|/n
= 15/2 = 7.5

Tracking Signal Computation

Mo	Forc	Act	Error	RSFE	Abs Error	Cum |Error|	MAD	TS
1	100	90	-10	-10	10	10	10.0	-1
2	100	95	-5	-15	5	15	7.5	-2
3	100	115						
4	100	100						
5	100	125						
6	100	140						

TS = RSFE/MAD
= -15/7.5 = -2

Plot of a Tracking Signal



Causal Models

- Causal models establish a cause-and-effect relationship between independent and dependent variables
- A common tool of causal modeling is linear regression:

$$Y = a + bx$$

Linear Regression

- Identify dependent (**y**) and independent (**x**) variables
- Solve for the slope of the line

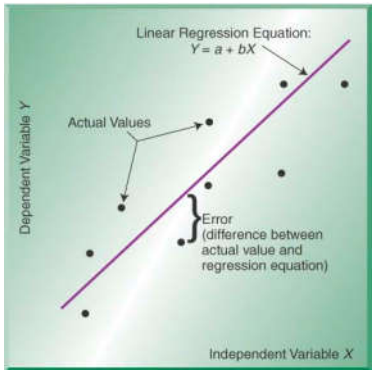
$$b = \frac{\sum XY - n\bar{X}\bar{Y}}{\sum X^2 - n\bar{X}^2}$$

- Solve for the y intercept

$$a = \bar{Y} - b\bar{X}$$

- Develop your equation for the trend line

Y=a + bX



Linear Regression Problem

- A maker of golf shirts has been tracking the relationship between sales and advertising expenditure. Use linear regression to find out what sales might be if the company invested Rs. 53 lacs in advertising next year.

	Adv. (X)	Sales (Y)
1	32	130
2	52	151
3	50	150
4	55	158
5	53	?

Linear Regression Problem

	Adv. (X)	Sales (Y)	XY	X^2
1	32	130	4160	1024
2	52	151	7852	2704
3	50	150	7500	2500
4	55	158	8690	3025
Sum	189	589	28202	9253
Average	47.25	147.25	7050.5	2313.25
b =	1.15182		a =	92.82649
5	53	153.75		

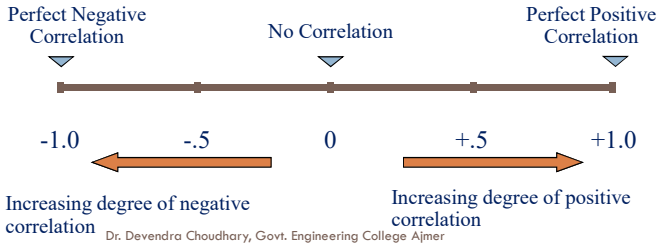
$$b = \frac{\sum XY - n\bar{X}\bar{Y}}{\sum X^2 - n\bar{X}^2}$$

$$a = \bar{Y} - b\bar{X}$$

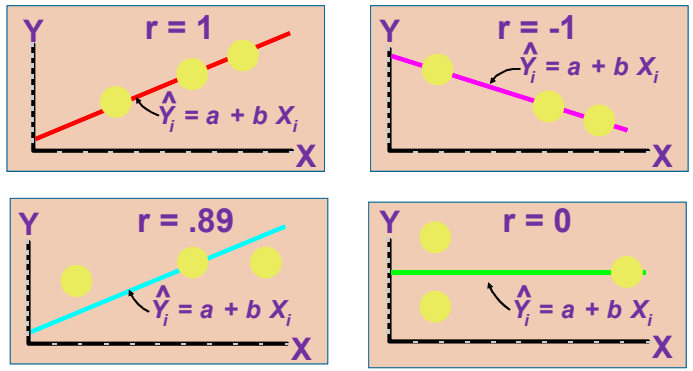
Y=a + bX
Y=92.8 + 1.15*53

Correlation

- Answers: '*how strong*' is the linear relationship between the variables?'
- Coefficient of correlation is denoted by **r**
  - Values range from -1 to +1
  - Measures degree of association
- Used mainly for understanding



Correlation



Regression and Linear Trend Projection

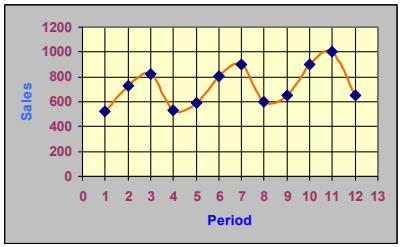
- Used for forecasting linear trend line
- Assumes relationship between response variable, *Yi* and time, *Xi* is a linear function

$$\hat{Y}_i = a + bX_i$$

- Estimated by least squares method
  - Minimizes sum of squared errors
- Proceed same way as we solved for causal model

Regression and Seasonal Index

Year	Q1	Q2	Q3	Q4
1	520	730	820	530
2	590	810	900	600
3	650	900	1000	650



Regression and Seasonal Index

Year	Q1	Q2	Q3	Q4
1	520	730	820	530
2	590	810	900	600
3	650	900	1000	650
Sum	1760	2440	2720	1780

Regression and Seasonal Index

Year	Q1	Q2	Q3	Q4	Annual total
1	520	730	820	530	2600
2	590	810	900	600	2900
3	650	900	1000	650	3200
Sum	1760	2440	2720	1780	8700
Avg	586.7	813.3	906.7	593.3	725.0

Regression and Seasonal Index

Year	Q1	Q2	Q3	Q4	Annual total
1	520	730	820	530	2600
2	590	810	900	600	2900
3	650	900	1000	650	3200
Sum	1760	2440	2720	1780	8700
Avg	586.7	813.3	906.7	593.3	725.0
SI	0.809	1.122	1.251	0.818	

Regression and Seasonal Index

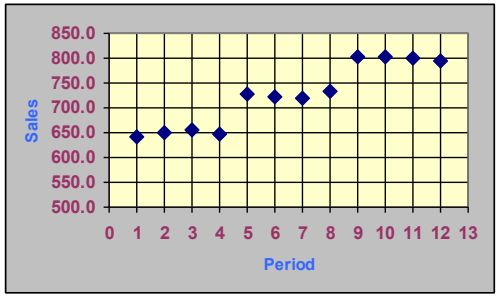
De-seasonalized data

For year 1 and Q1, 520 / 0.809 = 642.6

Year	Q1	Q2	Q3	Q4
1	642.6	650.7	655.7	647.6
2	729.1	722.0	719.7	733.1
3	803.3	802.3	799.6	794.2

Regression and Seasonal Index

De-seasonalized data



Regression and Seasonal Index

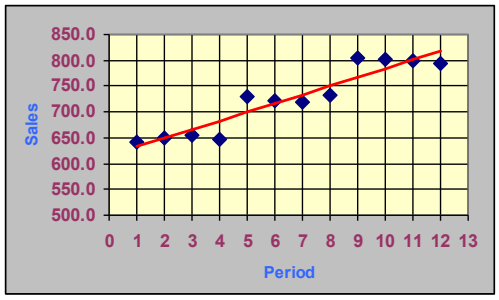
$Y = 615.41 + 16.86\ x$

$r = 0.94$

x	y	x^2	y^2	xy
1	642.6	1	412952.3	642.6
2	650.7	4	423432.9	1301.4
3	655.7	9	429940.6	1967.1
4	647.6	16	419401.8	2590.4
5	729.1	25	531615.0	3645.6
6	722.0	36	521325.4	4332.2
7	719.7	49	517923.6	5037.7
8	733.1	64	537503.2	5865.2
9	803.3	81	645237.9	7229.4
10	802.3	100	643611.6	8022.5
11	799.6	121	639411.9	8796.0
12	794.2	144	630819.7	9530.9
Sum 78	8700	650	6353176	58961.01

Regression and Seasonal Index

De-seasonalized data



Regression and Seasonal Index

Deseasonalized Forecast

$Y = 615.41 + 16.86\ x$

Quarter	Prediction
13	834.6
14	851.5
15	868.3
16	885.2

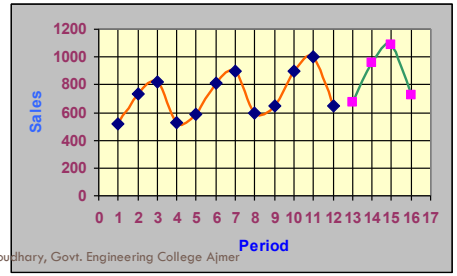
Regression and Seasonal Index

Quarter	Prediction	SI	Seasonal Forecast
13	834.6	0.809	675.3
14	851.5	1.122	955.2
15	868.3	1.251	1085.9
16	885.2	0.818	724.4

Regression and Seasonal Index

Year	Q1	Q2	Q3	Q4
1	520	730	820	530
2	590	810	900	600
3	650	900	1000	650
4	675	955	1086	724

Next Year's Forecast



Choosing a Forecasting Technique

- No single technique works in every situation
- Two most important factors
  - Cost
  - Accuracy
- Other factors include the availability of:
  - Historical data
  - Computers
  - Time needed to gather and analyze the data
  - Forecast horizon