QUALITY MANAGEMENT

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Learning objectives

When you complete this unit you should be able to

- Define the terms reliability, maintainability and availability
- Understand the importance of reliability, maintainability and availability
- Understand the concepts of failure and its causes
- Differentiate among MTTF, MTBF and MTTR
- Define lifetime related functions such as density, failure, survival and hazard rate
- Calculate lifetime functions from time to failure data
- Analyze the failure data for exponential and Weibull distributions

Learning objectives

When you complete this unit you should be able to

- Sketch the bathtub curve and describe its three phases
- Calculate system-wide reliability of series, parallel, mixed and *m*-out-of-*n* systems
- Explain various reliability improvement techniques
- Explain the concept of redundancy
- Differentiate between active and standby redundant systems
- Differentiate between low-level and high-level redundancies
- Calculate the maintainability and the availability

Reliability & Quality

- While **quality** is conformance of customer requirement only at a given point of time, the **reliability** is conformance of customer requirement over a period of time.
- Reliability is quality over time.

Reliability Definition

Reliability is the **probability** that a product, equipment or process will perform its intended **function**, without **failure**, under specific **conditions** for a specific period of **time**.

Reliability Definition

- **Probability:** Reliability is a probability, a probability of performing without failure; thus, reliability is a number between zero and one.
- **Failure:** A failure is defined as any functioning of the device or component which is not considered within the prescribed limits of satisfactory functioning. For example, if the function of a pump is to deliver at least 500 liters of fluid per minute and it is now delivering 400 liters per minute, the pump has failed, by this definition.
- **Function:** The device whose reliability is in question must perform a specific function. For example, if a blade of lawn-mower used to trim hedges breaks, this should not be charged as a failure as it can be repaired or replaced.

Reliability Definition

- **Conditions:** The device must perform its function under given conditions. For example, if electrical generators intended for use in ambient temperatures of 0-120 degrees Fahrenheit are brought to operate in the below zero degree Fahrenheit and fail, we should not charge failures to these units.
- **Time:** The device must perform for a period of time. One should never cite a reliability figure without specifying the time in question. The exception to this rule is for one-shot devices such as amunitions, rockets, automobile air-bags.

Non-repairable system

- A non-repairable system (or component, unit, part, etc.) is discarded upon first failure.
- For example, bulb, satellite, microchips and many electronic circuits.
- The lifetime of non-repairable system *T* is a random variable and described by a single time to failure (TTF).
- By noting values of TTF for a large number of identical and independent items, one can fit an appropriate distribution.
- For most of the items, the TTF assumes exponential or Weibull distributions.
- For given TTF distribution, one can determine mean time to failure (MTTF).
- The average time to failure of the non-repairable system after entering into service is known as mean time to failure (MTTF).

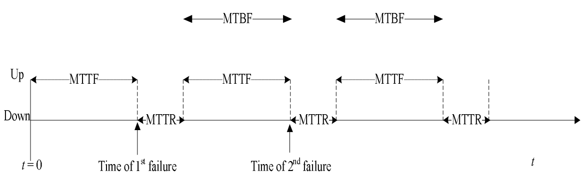
Repairable system

- A repairable system is a system which, upon failure, is restored to operation by any repair action other than replacing the entire system.
- For example, autos and appliances can be repaired.
- There are two random variables of interest in case of repairable systems.
  - The number of survivals or failures at a given point of time *t*
  - The operating time between successive failures (TBF)
- The mean operating time between failures (abbreviated as MTBF) is the expected length of time between successive failures of a repairable component or system.

Relationship among MTTF, MTBF and MTTR

- **MTTF** is reserved for a **non-repairable component** or system, **MTBF** is used as a reliability measure for the study of **repairable systems**.
- Sometimes, MTTF is also used for repairable systems, where it represents a mean time to first failure of the item.
- The term **MTTR is mean time to repair**. It is defined as the mean time elapsed in restoring a process or system for stated operational condition after a failure through maintenance and repair.

Relationship among MTTF, MTBF and MTTR



Product lifetime related functions

- These functions are widely used in measuring reliability.
- Let *T* be the random variable defining the lifetime of the product, which is the time the product will operate before failure.
  - Sometimes *T* can take only discrete countable values, for example, a number of cycles.
  - Most of the time, however, time to failure *T* will be a continuous random variable.

Failure density function

- Failure density function $f(t)$ is the probability density function (PDF) of the TTF.
- It approximates the probability of failure in a small time interval Δt or probability of failure around an age of t. That is

$$f(t) = \lim_{\Delta t \to 0} \frac{P\left(t - \frac{\Delta t}{2} < T \leq t + \frac{\Delta t}{2}\right)}{\Delta t}$$

- For a product starting at age $t = 0$, the probability to fail up to an age $t > 0$ is given by

$$P(T \leq t) = \int_0^t f(t)dt$$

Failure density function

- The PDF of TTF satisfies below given two conditions:
  - $f(t) \geq 0$, for all t
  - $\int_0^\infty f(t)dt = 1$
- The probability of reaching an age between t_1 and t_2, $t_1 < t_2$, is

$$P(t_1 \leq T \leq t_2) = \int_{t_1}^{t_2} f(t)dt$$

Failure density function

- Assume that there are a large number of independent and identical items that started operating in a common environment at $t = 0$, $N(0) = N$. Failure times of items are recorded, and therefore the number of operating items $N(t)$ at each instant of time $t \geq 0$ is known. Then

$$f(t) = \frac{\text{Number of failures during a unit interval}}{\text{Total number of items}}$$

- If n_k is number of failures during k^{th}, Δt interval, we have

$$f(t_k) = \frac{n_k}{N\Delta t}$$

Failure (or unreliability) function

- The probability that a process, device or system will not perform its intended function for a given interval of time under specified operating conditions is called **unreliability**.
- The **failure distribution function $F(t)$**, also known as unreliability, gives the probability of failing up to age t or of having a life span of at most length t, that is

$$F(t) = P(T \leq t)$$

- The value of failure function increases as the time increases.

Failure (or unreliability) function

- $F(t)$ is identical to the cumulative distribution function (CDF) in probability theory and hence gives the probability that a measured value of TTF will fall between 0 and t.
- In probability theory, CDF and PDF of a lifetime variable are related as follows:

$$f(t) = \lim_{\Delta t \to 0} \frac{F(t + \Delta t) - F(t)}{\Delta t} = \frac{dF(t)}{dt} \quad \text{Or} \quad F(t) = \int_0^t f(t)dt$$

- Thus, the failure function is equal to the area beneath the probability density function.

Failure (or unreliability) function

- In case of testing a large number of independent and identical items N, if the number of operating items $n(t)$ and the number of failed items $m(t)$ at each instant of time $t \geq 0$ is known. Then, $F(t)$ is the fraction of items that fail by time t. That is

$$F(t) = \frac{m(t)}{N} = \frac{m(t)}{m(t) + n(t)}$$

Reliability (or Survival) function

- The reliability function is the probability of no failures in the interval between 0 and t or equivalently, the probability of failure after time t, that is

$$R(t) = P(T > t) \qquad t \ge 0$$

- In case of testing a large number of independent and identical items N, if the number of operating items $n(t)$ and the number of failed items $m(t)$ at each instant of time $t \ge 0$ is known. Then, $R(t)$ is the fraction of items in a population that survive up to time t. That is

$$R(t) = \frac{n(t)}{N} = \frac{n(t)}{m(t) + n(t)}$$

Relationship between $R(t)$ and $F(t)$

- The failure function and the reliability function are complementary functions, so

$$R(t) = 1 - F(t) = 1 - \int_0^t f(t)dt = \int_t^\infty f(t)dt$$

- $R(t)$ is the probability of exceeding t and $F(t)$ is the probability of reaching t.
- In other words, $R(t)$ gives the probability of its functioning at time t and $F(t)$ is the probability of its being down at time t.

Hazard rate function

- It quantifies the risk of failure as the age of the system increases.
- The hazard rate function is the conditional probability of a failure in time interval from t to $t + \Delta t$ given that the system has survived to time t, that is

$$h(t) = P(t \le T \le t + \Delta t / T \ge t) = \lim_{\Delta t \to 0} \frac{F(t+\Delta t) - F(t)}{R(t)\Delta t} = \lim_{\Delta t \to 0} \frac{R(t) - R(t+\Delta t)}{R(t)\Delta t}$$

- Note that hazard function $h(t)$ is not a probability and hence can be greater than 1.

Hazard rate function

$$h(t) = \lim_{\Delta t \to 0} \frac{F(t+\Delta t) - F(t)}{R(t)\Delta t} = \frac{f(t)}{R(t)} = -\frac{dR(t)}{dt} \cdot \frac{1}{R(t)}$$

- The hazard rate function is the ratio of the probability density function to the reliability function.
- Integrating both sides of the above equation, we get:

$$\int_0^t h(t)dt = -\int_0^t \frac{dR(t)}{R(t)} \quad \text{or} \quad \int_0^t h(t)dt = -\ln R(t) \quad \text{or} \quad R(t) = \exp\left(-\int_0^t h(t)dt\right) = e^{-\int_0^t h(t)dt}$$

Failure rate function

- The hazard function can be increasing, decreasing or constant.
- Whenever, the hazard function is constant, we call it as failure rate λ.

$$\lambda = \frac{\text{Total number of failures in a population}}{\text{Cumulative operating time of the population}}$$

Failure rate function

- Assume that there are N number of independent and identical items that started operating in a common environment at $t = 0$. If after $t \ge 0$, the number of operating items remains $N(t)$. Then

$$\lambda = \frac{1}{t} \cdot \frac{N - N(t)}{N}$$

- For example, if 1000 controllers that started operating in a common environment at $t = 0$. After 10 hours if the number of operating controllers remains 950, then λ = 50/(1000*10) = 0.005 failures per hour.

MTTF

- The mean time to failure (MTTF) is mean lifespan or expected time to failure of a component or system.

$$\text{MTTF} = E(t) = \int_0^\infty t f(t)dt = \int_0^\infty t\left(-\frac{dR(t)}{dt}\right)dt = -tR(t)\big|_0^\infty + \int_0^\infty R(t)dt$$

Since, at $t = 0$, $R(t) = 1$ and at $t = \infty$, $R(t) = 0$, so

$$\text{MTTF} = \int_0^\infty R(t)dt$$

- Thus, the mean life or MTTF is equal to the area beneath the reliability function.

Expected number of failures

- If $N(t)$ is the total number of failures by time t, then the expected number of failures $E[N(t)]$ by time t can be obtained by taking integration of the hazard function from period 0 to t. That is

$$E[N(t)] = \int_0^t h(t)dt$$

Example 1

- Consider that one hundred identical products are installed and the number of products that fail during each year interval is noted. Total number of failed products at the end of 1 year, 2 year, 3 year and so on is given below.

Time interval	1	2	3	4	5	6	7	8	9	10	11
Number of failures	22	16	12	10	8	7	5	4	4	3	9

- Compute failure density, failure function, hazard rate, reliability and MTTF.

Example 1

- Let N be the total initial population and n_k be the number of failures during k^{th} unit interval, the failure density is obtained using Equation

$$f(t_1) = \frac{n_1}{N\Delta t} = \frac{22}{100} = 0.22 \qquad f(t_2) = \frac{n_2}{N\Delta t} = \frac{16}{100} = 0.16$$

- Since, failure function is the cumulative distribution function of time to failure, we have

$$F(t_k) = \sum_{i=1}^{k} f(t_i) = f(t_1) + f(t_2) + \dots + f(t_i)$$

- For example,

$$F(t_1) = 0.22 \qquad F(t_2) = 0.22 + 0.16 = 0.38$$

Example 1

- As the failure function and the reliability function are complementary functions, so

  $R(t_k) = 1 - F(t_k)$

- For example,

  $R(t_1) = 1 - F(t_1) = 1 - 0.22 = 0.78$ and

  $R(t_2) = 1 - F(t_2) = 1 - 0.38 = 0.62$

Example 1

- Hazard rate is the ratio of the number of failures during k^{th} unit interval to the average population in that particular interval, that is

$$h(t_k) = \frac{n_k}{\left(N(t_{k-\frac{\Delta t}{2}}) + N(t_{k+\frac{\Delta t}{2}})\right)/2}$$

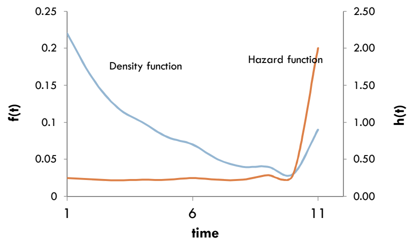
- For example

$$h(t_1) = \frac{22}{(100+78)/2} = 0.2472 \qquad h(t_2) = \frac{16}{(78+62)/2} = 0.2286$$

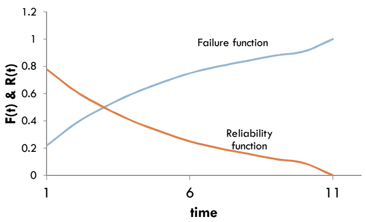
Example 1

Interval	Number of failures	Cumulative failures	Number of survivors	Failure density	Failure function	Hazard rate	Reliability function
1	22	22	78	0.22	0.22	0.2472	0.78
2	16	38	62	0.16	0.38	0.2286	0.62
3	12	50	50	0.12	0.5	0.2143	0.50
4	10	60	40	0.10	0.6	0.2222	0.40
5	8	68	32	0.08	0.68	0.2222	0.32
6	7	75	25	0.07	0.75	0.2456	0.25
7	5	80	20	0.05	0.8	0.2222	0.20
8	4	84	16	0.04	0.84	0.2222	0.16
9	4	88	12	0.04	0.88	0.2857	0.12
10	3	91	9	0.03	0.91	0.2857	0.09
11	9	100	0	0.09	1.00	2.0000	0.00

Example 1



Example 1



Example 1

- **Let n_k (k = 1, 2, ..., l) be the number of failures during k^{th}, Δt interval, then mean time to failure will be**

$$\text{MTTF} = \frac{1}{N}\sum_{k=1}^{l} k n_k = \frac{1}{N}(n_1 + 2n_2 + 3n_3 + \dots + k n_k)$$

- **In given problem, we have**

MTTF = (1*22 + 2*16 + 3*12 + 4*10 + 5*8 + 6*7 + 7*5 + 8*4 + 9*4 + 10*3 + 11*9) / 100

MTTF = 444 / 100 = 4.44 years

Example 2

- **The failure density function for a class of components is given by**

$$f(t) = 0.25 - \left(\frac{0.25}{8}\right)t$$

where t is in years. Find failure distribution, reliability and hazard rate functions. Sketch the four functions and also find MTTF.

Example 2

- **Given,** $f(t) = 0.25 - \left(\frac{0.25}{8}\right)t$
- **Failure distribution function is given by**

$$F(t) = \int_0^t f(t)\,dt = \int_0^t \left(0.25 - \left(\frac{0.25}{8}\right)t\right)dt = 0.25\,t - \left(\frac{0.25}{16}\right)t^2$$

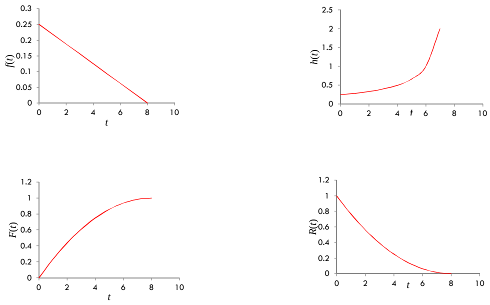
- **We know,**

$$R(t) = 1 - F(t) = 1 - 0.25\,t + \left(\frac{0.25}{16}\right)t^2$$

- **Since, the hazard rate function is the ratio of the probability density function to the reliability function, that is**

$$h(t) = \frac{f(t)}{R(t)} = \frac{0.25 - \left(\frac{0.25}{8}\right)t}{1 - 0.25\,t + \left(\frac{0.25}{16}\right)t^2} = \frac{2 - 0.25t}{8 - 2t + 0.125t^2}$$

Example 2



Example 2

- Integrate reliability function from 0 to 8 to get MTTF as it is equal to the area under the reliability function, that is

$$\text{MTTF} = \int_0^8 \left(1 - 0.25\,t + \left(\frac{0.25}{16}\right)t^2\right)dt = \left[t - \frac{0.25}{2}t^2 + \frac{0.25}{48}t^3\right]_0^8 = 2.667\,\text{years.}$$

Example 3

- The hazard rate function for a class of components is given by

$$h(t) = 3t^2 - 2t$$

where t is in hours. Find failure density and reliability functions. Also find reliability at t = 2.

Example 3

- Given, $h(t) = 3t^2 - 2t$
- Following equation is used to derive the reliability function from a known hazard rate function.

$$R(t) = \exp\left(-\int_0^t h(t)dt\right) = e^{-\int_0^t h(t)dt}$$

$$R(t) = \exp\left(-\int_0^t \left(3t^2 - 2t\right)dt\right) = \exp\left(-\left(t^3 - t^2\right)\right) = e^{t^2(1-t)}$$

- At t = 2, $R(2)$ = 0.0183

Example 3

- Following equation is used to derive the failure density function from a known reliability function or hazard rate function.

$$f(t) = h(t)R(t) = h(t)\exp\left(-\int_0^t h(t)dt\right) = \left(3t^2 - 2t\right)e^{t^2(1-t)}$$

Lifetime functions under exponential failure distribution

- The failure density function under exponential distribution is given by:

$$f(t) = \lambda e^{-\lambda t}$$

- The remaining lifetime functions under exponential distribution are given by:

$$F(t) = 1 - e^{-\lambda t} \qquad R(t) = e^{-\lambda t}$$

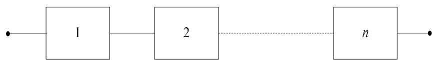
$$h(t) = \lambda \qquad \text{MTTF} = \frac{1}{\lambda} \qquad E[N(t)] = \lambda t$$

System reliability models

- A system consists of a complex configuration of multiple components.
- System reliability models are usually studied to determine:
  - The reliability of a system for given configuration of several components, or
  - The number of components and their structure to achieve target reliability.

Series system

- In series reliability system, all the components must be working for the system to function as these may be connected in series.



- The word series does not imply the physical arrangement of the components; rather it describes the response of the system to the failure of one of its components.

Series system

- If there are *n* (*i* = 1, 2, …, *n*) components in series in a system and $R_i(t)$ be the corresponding reliabilities, then the reliability of system $R_s(t)$ is given by:

$$R_s(t) = \prod_{i=1}^{n} R_i(t) = R_1(t) * R_2(t) * \ldots * R_n(t)$$

- The reliability of a series system is the product of the component reliabilities.

Series system

- If the components have exponential failure distribution, then system reliability reduces to:

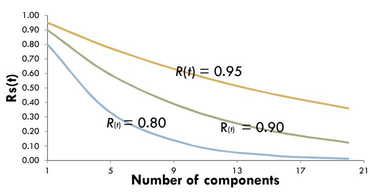
$$R_s(t) = e^{-\lambda_1 t} * e^{-\lambda_2 t} * \ldots * e^{-\lambda_n t} = e^{-\left(\sum_{i=1}^{n} \lambda_i t\right)}$$

- If λ_s is the failure rate of a series system, you can note from the above equation that it is the sum of the component failure rates. That is

$$\lambda_s = \sum_{i=1}^{n} \lambda_i \quad \text{and} \quad \mathrm{MTTF}_s = \frac{1}{\sum_{i=1}^{n} \lambda_i} = \frac{1}{\lambda_s}$$

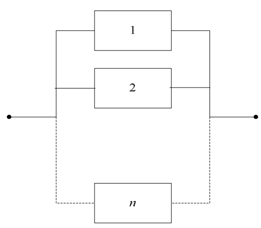
Series system

- In a series system, the system reliability is a function of
  - Individual component reliabilities and
  - The number of components in series.



Parallel system

- In parallel reliability system, at least one of the components must be working for the system to function as these are connected in parallel.



Parallel system

- If there are n (i = 1, 2, …, n) components in parallel in a system and $R_i(t)$ be the corresponding reliabilities, then the reliability of parallel system $R_s(t)$ is determined by first calculating the probability that system will fail. That is

$$R_s(t) = 1 - F_s(t) = 1 - \prod_{i=1}^{n} F_i(t) = 1 - \prod_{i=1}^{n} (1 - R_i(t))$$

$$R_s(t) = 1 - \{(1 - R_1(t)) * (1 - R_2(t)) * \ldots * (1 - R_n(t))\}$$

Parallel system

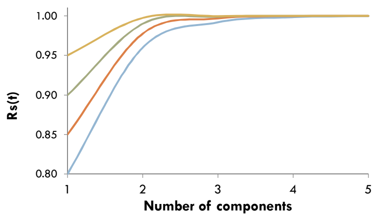
- If similar components (i.e., $\lambda_1 = \lambda_2 = \ldots = \lambda_n = \lambda$), having exponential time to failure distribution, are connected in parallel, then system reliability and mean time to failure are given by:

$$R_s(t) = 1 - \left(1 - e^{-\lambda t}\right)^n$$

$$\text{MTTF}_s = \frac{1}{\lambda} \sum_{i=1}^{n} \frac{1}{i}$$

Parallel system

- The reliability of a parallel system **increases with increase in the number of components, or increase in individual component reliabilities**, or both.



Example 4

- Three components X, Y and Z have reliabilities of 0.92, 0.95 and 0.96 respectively. Compare the reliability of a series system with parallel system made up of these components.
- **Solution**
- Given: $R_X(t) = 0.92$, $R_Y(t) = 0.95$ and $R_Z(t) = 0.96$
- The reliability of a series system is the product of the component reliabilities, that is

$$R_s(t)_{\text{Series}} = R_X(t) * R_Y(t) * R_Z(t)$$

$$R_s(t)_{\text{Series}} = 0.92 * 0.95 * 0.96 = 0.839$$

Example 4

- The reliability of parallel system is given by

$$R_s(t)_{\text{Parallel}} = 1 - \{(1 - R_X(t)) * (1 - R_Y(t)) * (1 - R_Z(t))\}$$

$$R_s(t)_{\text{Parallel}} = 1 - \{(1 - 0.92) * (1 - 0.95) * (1 - 0.96)\} = 0.9998$$

- We observe that the system-wide **reliability of parallel system is higher than series system**.
- Therefore, a higher reliability can be achieved by connecting components in parallel.

Example 5

- A series system is composed of four components with failure rates of 0.002, 0.001, 0.0025 and 0.0005. What is the 100 hours system reliability? Also, compute MTTF.
- **Solution**
- The series system reliability is given by

$$R_s(t) = e^{-\lambda_1 t} * e^{-\lambda_2 t} * \ldots * e^{-\lambda_n t} = e^{-\left(\sum_{i=1}^{n} \lambda_i t\right)}$$

Example 5

- For given problem, we have

$$R_s(t) = e^{-(0.002+0.001+0.0025+0.0005)t} = e^{-0.006\,t}$$

- At t = 100 hours, we have

$$R_s(t) = e^{-0.006\,t} = e^{-0.006*100} = 0.5488$$

$$\text{MTTF}_s = \frac{1}{\sum_{i=1}^{n}\lambda_i} = \frac{1}{0.006} = 166.67 \quad \text{hours}$$

Example 6

- The reliability of a communication channel is 0.60. How many identical channels should be placed in parallel so as to achieve the reliability of communication system as 0.93?
- Solution
- The reliability of parallel system is given by

$$R_s(t) = 1 - \prod_{i=1}^{n}(1 - R_i(t))$$

Example 6

- We rearrange above equation as follows:

$$\prod_{i=1}^{n}(1 - R_i(t)) = 1 - R_s(t)$$

- As components are identical, we can write

$$(1 - R_i(t))^n = 1 - R_s(t) \quad \text{or} \quad n = \frac{\ln(1 - R_s(t))}{\ln(1 - R_i(t))}$$

- For given $R_s(t)$ = 0.93 and $R_i(t)$ = 0.60

$$n = \frac{\ln(1 - 0.93)}{\ln(1 - 0.60)} = 2.90 \approx 3$$

- Therefore, three channels should be placed in parallel to achieve desired reliability.

Reliability improvement techniques

- There are several ways to improve system reliability:
  - Product design
  - Redundancy
  - Maintenance
- The performance of maintenance is measured in terms of maintainability and availability.

Design for reliability

- Design for reliability is a process which is performed during the design of the product so as to ensure that the product is able to perform to a required level of reliability.

- The important product characteristic like failure rate, failure mode, failure mechanism, availability, life of the product, maintenance ease are used in developing the product design.

Design for reliability

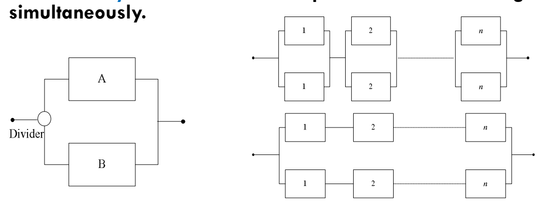
- The following activities are performed to ensure design for reliability:
  - Specify product reliability targets before any design work is undertaken.
  - Include reliability requirements as per likely service conditions in the problem definition.
  - Designs are to be assessed based on ease of inspection, ease of maintenance and cost of maintenance.
  - Select reliable parts and components in new product design based on estimating failure rate, failure mode, MTTF and others.
  - Check for quality assurance during production.
  - Take feedback related to service failures, MTTF and MTTR, which is to be used to improve the design.

Redundancy

- In reliability engineering, redundancy refers to the **use of more than one component** or system for the **same function**.
- The increase in the reliability value depends on:
  - reliability values of the individual components,
  - number of components and
  - type of configuration in which these components are connected with one another.

Active (or dynamic) redundancy

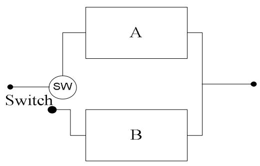
- The parallel reliability systems are often called active redundant systems as all the components are functioning simultaneously.



- The component level redundancy is known as low-level redundancy, whereas the system level redundancy known as high-level redundancy.

Standby (or passive) redundancy

- A standby unit cuts in and takes over when the current operating unit fails.
- The unit should be provided by sensors and switching mechanisms to sense the failure and to place the unit in service.



Maintenance

- Maintenance includes activities such as cleaning, lubrication, topping up, adjustment and calibration, condition assessment, repairs and replacement.
- Maintenance enables to detect and prevent failures as they would occur and hence increase system reliability and availability.
- The effectiveness of maintenance is mainly assessed by maintainability and availability measures.

Maintainability

- Maintainability $M(t)$ is the probability that a system that has failed can be retained in or restored to a specified operable condition within a specified interval of time, when maintenance is performed in accordance with prescribed procedures.
- Maintainability is a characteristic of design, installation, and operation of system and equipment.
- It is quantified by mean time to repair (MTTR).

Maintainability

- Assume that an equipment has m failures during a certain period of time selected for our analysis.
- Obviously, time to repair (TTR) these failures would be a random variable. Let $g(t)$ be the probability density function of TTR, then

$$\mathrm{MTTR} = \frac{\sum_{i=1}^{m} \mathrm{TTR}_i}{m} = \int_0^\infty t g(t) dt$$

$$M(t) = P(\mathrm{TTR} \le t) = \int_0^t g(t) dt$$

Maintainability

- For an exponential time to repair distribution, we have

$$M(t)=1-e^{-\frac{t_r}{\text{MTTR}}}=1-e^{-\mu t_r}$$

where μ = 1/MTTR is called repair rate.

- Let a system failed 50 times during its lifetime. Assume that maintenance hours used to repair these failures is 250 hours. Then, MTTR = 250/50 = 5 hours or μ = 0.2 repairs per hour.

Maintainability

- Maintainability for repair time 1 hour, 2 hours and 10 hours are computed below:

$$M(1)=1-e^{-0.2*1}=0.1812 \qquad M(2)=1-e^{-0.2*2}=0.3296$$

$$M(10)=1-e^{-0.2*10}=0.8646$$

- The interpretation of these results is as follows:
  - A failure has only 18% chance of being repaired in 1 hour; however 86% chance of being repaired in 10 hours.
  - In other words, 18 failures out of 100 will be repaired in 1 hour, but 86 can be repaired in 10 hours.

Availability

- An equipment (process or system) may be either in working state or in non-working state during its specified life.
- The running or working time of a system is called **up time**.
- The time period during which a system is not able to deliver requested services is called **down time**.

Availability

- The system may be down due to following reasons:
  - Equipment failures
  - Tooling damage
  - Unplanned maintenance
  - Process warm up
  - Machine changeovers
  - Material shortage
- The time elapsed in **setup, planned maintenance** and any **scheduled shut down** are not the part of the down time.

Availability

- **Operational availability** $A_o(t)$ is the proportion of the up time to the sum of up and down times. That is

$$A_o(t)=\frac{\text{Up time}}{\text{Up time}+\text{Down time}}$$

- Note that up time is MTBF/MTFF and down time is MTTR.

$$A_o(t)=\frac{\text{MTTF}}{\text{MTTF}+\text{MTTR}}$$

- If TTR and TTF distribution are exponential, then λ = 1/MTTF and μ = 1/MTTR and we get

$$A_o(t)=\frac{\mu}{\lambda+\mu}$$

- Operational availability is also called as **steady-state availability** or **inherent availability**.