Syllabus

Thermal Radiation: Plank distribution law, Krichoff's law; radiation properties, diffuse radiations; Lambert's law. Radiation intensity.

Heat exchange between two black bodies, heat exchanger between gray bodies.

Shape factor; electrical analogy; reradiating surfaces heat transfer in presence of reradiating surfaces.

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Radiation Exchange Between Surfaces

- The actual process of relative heat exchange between different surfaces takes place in the presence of either a non-participation of medium or a participating medium.
- Most of the gases meet the requirements of non-participating medium to an excellent approximation; exception are carbon dioxide and water vapors which have high absorptivity at certain wavelength of infrared radiation.

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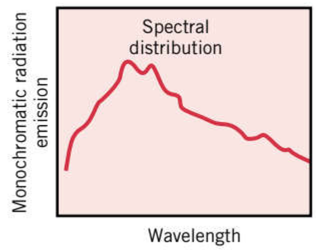
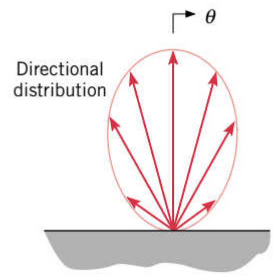
Radiation Intensity

- Radiation that leaves a surface can propagate in all possible directions
- Radiation incident upon a surface may come from different directions,
- The surface responds to this radiation depends on the direction.
- Such directional effects can be of primary importance in determining the net radiative heat transfer rate and may be treated by introducing the concept of radiation intensity.





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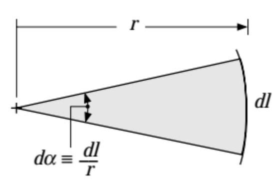
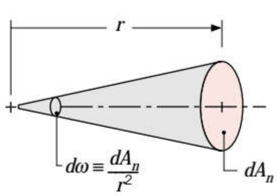
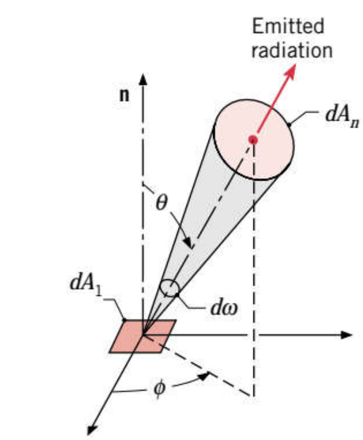
Radiation Intensity



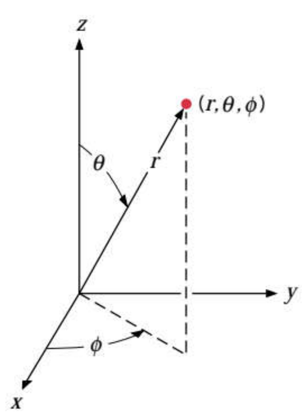
Plane angle



Solid angle



Emission of radiation from a differential area dA_1 into a solid angle $d\omega$ subtended by dA_n at a point on dA_1



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Derivation of View Factor for Radiation Exchange Between Surfaces A_1-A_2

- Each elemental area subtends a solid angle at the center of the other.
- Let $d\omega_1$ be subtended at dA_1 by dA_2 and $d\omega_2$ subtended at dA_2 by dA_1, Then

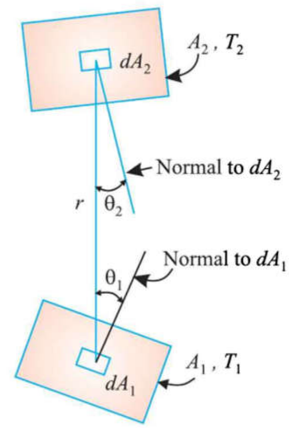
$$d\omega_1 = \frac{dA_2 \cos\theta_2}{r^2}, \quad \text{and} \quad d\omega_2 = \frac{dA_1 \cos\theta_1}{r^2}$$

- The energy leaving dA_1 in the direction given by the angle per unit solid angle

$$I_{b1} \cdot dA_1 \cdot \cos\theta_1$$

- The rate of radiant energy leaving dA_1 and striking on dA_2 is

$$dQ_{1-2} = I_{b1}\, dA_1 \cos\theta_1 \,.\, d\omega_1 = \frac{I_{b1} \cos\theta_1 \cos\theta_2\, dA_1\, dA_2}{r^2}$$



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Radiation Exchange Between Surfaces A1-A2

- The quantity of energy radiated by dA_2 and absorbed by dA_1 is given by:

$$dQ_{2\text{-}1} = \frac{I_{b2} \cos\theta_2 \cos\theta_1\, dA_2\, dA_1}{r^2}$$

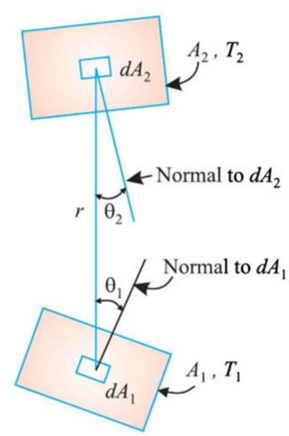
- The net rate of transfer of energy between dA_1 and dA_2 is

$$dQ_{12} = dQ_{1-2} - dQ_{2-1} = \frac{dA_1\, dA_2 \cos\theta_1 \cos\theta_2}{r^2}(I_{b1} - I_{b2})$$

$$I_{b1} = \frac{E_{b1}}{\pi} \text{ and } I_{b2} = \frac{E_{b2}}{\pi}$$

$$dQ_{12} = \frac{dA_1\, dA_2 \cos\theta_1 \cos\theta_2}{\pi r^2}(E_{b1} - E_{b2})$$

$$dQ_{12} = \frac{\sigma\, dA_1\, dA_2 \cos\theta_1 \cos\theta_2}{\pi r^2}(T_1^4 - T_2^4)$$



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Radiation Exchange Between Surfaces A1-A2

- The rate of total net heat transfer for the total areas A_1 and A_2 is given by:

$$Q_{12} = \int dQ_{12} = \sigma (T_1^4 - T_2^4) \int_{A_1} \int_{A_2} \frac{\cos\theta_1 \cos\theta_2 \, dA_1 \, dA_2}{r^2} .$$

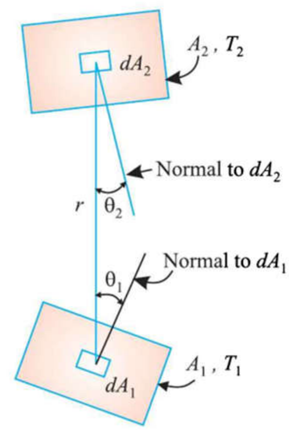
- The rate of radiant energy emitted by A1 that falls on A2

$$dQ_{1-2} = I_{b1} \, dA_1 \cos\theta_1 \, . \, d\omega_1$$

$$= \frac{I_{b1} \cos\theta_1 \cos\theta_2 \, dA_1 \, dA_2}{r^2}$$

$$Q_{1-2} = I_{b1} \int_{A_1} \int_{A_2} \frac{\cos\theta_1 \cos\theta_2 \, dA_1 \, dA_2}{r^2}$$

$$Q_{1-2} = \sigma T_1^4 \int_{A_1} \int_{A_2} \frac{\cos\theta_1 \cos\theta_2 \, dA_1 \, dA_2}{\pi r^2}$$



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Radiation Exchange Between Surfaces A1-A2

- The rate of total energy radiated by A1

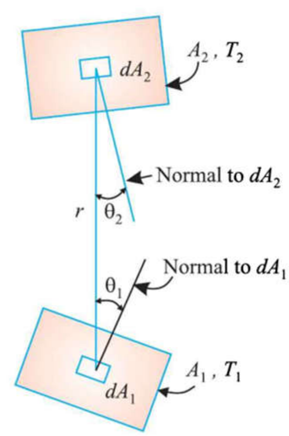
$$Q_1 = A_1 \sigma T_1^4$$

- Hence the fraction of the rate of energy leaving area A1 and impinging on area A2 is

$$\frac{Q_{1-2}}{Q_1} = \frac{1}{A_1} \int_{A_1} \int_{A_2} \frac{\cos\theta_1 \cos\theta_2 \, dA_1 \, dA_2}{\pi r^2}$$

$$\frac{Q_{1-2}}{Q_1} = F_{1-2}$$

- F_{1-2} is known as 'configuration factor' or 'shape factor' or 'view factor' between the two radiating surfaces and is a function of geometry only.
- Thus, the shape factor may be defined as" The fraction of radiative energy that is diffused from one surface element and strikes the other surface directly with no intervening reflections."



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The View Factor

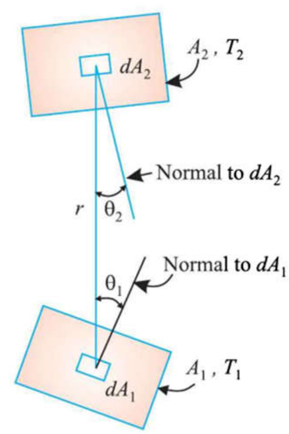
- Fraction of Energy emitted by A_1 and received by A_2

$$q_{A1-A2} = F_{A_1-A_2} A_1 G_1$$

- Fraction of Energy emitted by A_2 and received by A_1
- $q_{A2-A1} = F_{A_2-A_1} A_2 G_2$
- Where $F_{A1\text{-}A2}$ is fraction of radiation energy leaving A_1 that is incepted/received by A_2

$$F_{A_1-A_2} = \frac{1}{A_1} \int_{A_1} \int_{A_2} \frac{\cos\theta_1 \cos\theta_2}{\pi R^2} dA_2 dA_1$$

- And fraction of radiation from A2 to A1
- $F_{A_2-A_1} = \frac{1}{A_2} \int_{A_2} \int_{A_1} \frac{\cos\theta_2 \cos\theta_1}{\pi R^2} dA_1 dA_2$



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The View Factor

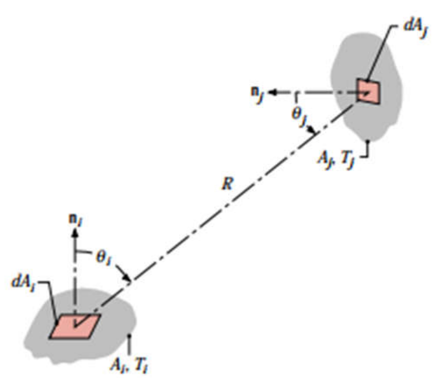
- From both equations of fraction of energy
- $F_{A_1-A_2} = \frac{1}{A_1} \int_{A_1} \int_{A_2} \frac{\cos\theta_1 \cos\theta_2}{\pi R^2} dA_2 dA_1$
- $F_{A_2-A_1} = \frac{1}{A_2} \int_{A_2} \int_{A_1} \frac{\cos\theta_2 \cos\theta_1}{\pi R^2} dA_1 dA_2$
- It seems that

$$F_{A_1-A_2} A_1 = F_{A_2-A_1} A_2$$

Or

$$F_{12} A_1 = F_{21} A_2$$

Reciprocity Relation



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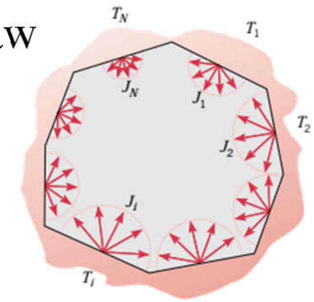
The View Factor: Enclosure Law



- The enclosure law state that
- $F_{11} + F_{12} + F_{13} + F_{14} + F_{15} + \cdots = 1$
- So, it can be say that

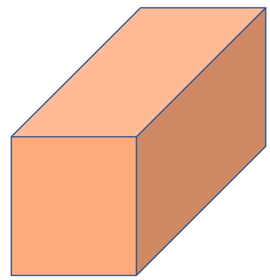
$$\sum_{j=1}^{N} F_{ij} = 1$$

- No. of View Factors = N^2
- Considering F_{11}

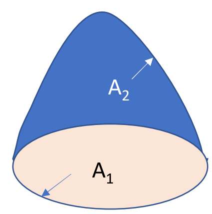
Plane surface

Convex surface

Concave surface



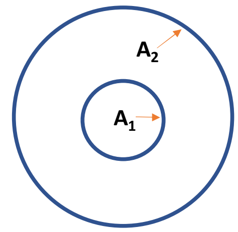
The View Factor: Example



- N=2 so VFs are 4
- Matrix of VF = $\begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}$
- As F_{11}+ F_{12} = 1 and F_{21}+ F_{22} = 1
- By observation, A1 is plane surface, so F_{11} =
- So F_{12} =
- From reciprocity relation F_{21} =
- And by law of enclosure F_{22} = 1 – F_{21}

The View Factor: Example

- By observation:
  - As A_1 is convex surface, so $F_{11} = 0$
  - All radiating energy leaving from A_1 directly received by A_2 : Yes, and $F_{11} + F_{12} = 1$. Therefore
  - $F_{12} = 1$
- From reciprocity relation
  - $F_{21} = (A_1/A_2)\ F_{12}$
  - Hence, $F_{22} = 1 - F_{21}$



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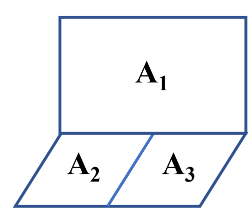
The View Factor: Example

- No. of view factors are: $N^2 = 3^2 = 9$
- View factor matrix: $\begin{bmatrix} F & F & F \\ F & F & F \\ F & F & F \end{bmatrix}$
- Solution
  - By observation
  - By Reciprocity relation
  - By Enclosure law
- $A_1\ F_{1\text{-}2,3} = A_1\ F_{1\text{-}2} + A_1\ F_{1\text{-}3}$
- $F_{1\text{-}2,3} = F_{1\text{-}2} + F_{1\text{-}3}$
- $A_{2,3\text{-}1}\ F_{2,3\text{-}1} = A_2 F_{2\text{-}1} + A_3 F_{3\text{-}1}$



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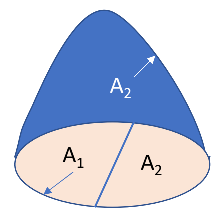
The View Factor: Example

- No. of view factors are: $N^2 = 3^2 = 9$
- View factor matrix: $\begin{bmatrix} F & F & F \\ F & F & F \\ F & F & F \end{bmatrix}$
- Solution
  - By observation: A_1 and A_2 are plane surface
    - So, $F_{11} = F_{22} = 0$
    - And, $F_{12} = F_{21} = 0$
  - By Reciprocity relation
    - $A3\ F_{31} = A_1\ F_{13}$ ➔ $F_{31} = (A_1/A_3)\ F_{13}$
  - By Observation $F_{32} = F_{13}$
  - By Enclosure law
    - $F_{33} = 1 – F_{31} – F_{32} = 1\text{-}2F_{31}$



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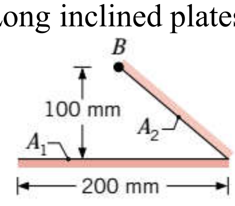
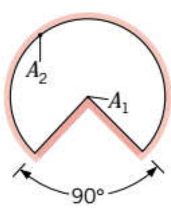
Find F_{12} and F_{21}?

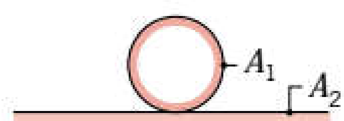
Long inclined plates



Long duct



Sphere lying on infinite plane



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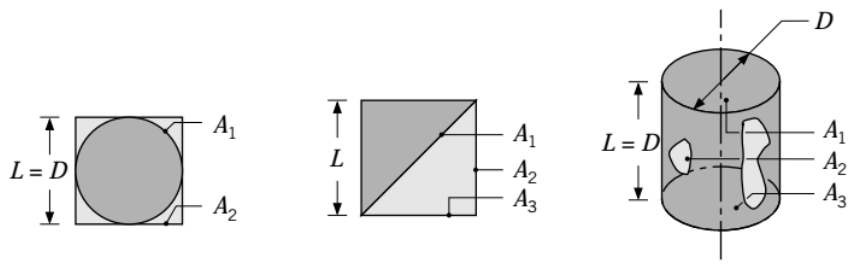
Find F_{12} and F_{21}?

1. Sphere of diameter D inside a cubical box of length L= D.
2. One side of a diagonal partition within a long square duct.
3. End and side of a circular tube of equal length and diameter.



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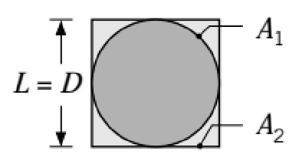
Find F_{12} and F_{21}?

1. Sphere within a cube:

By inspection, $F_{12} = 1$

By reciprocity, $F_{21} = \frac{A_1}{A_2} F_{12} = \frac{\pi D^2}{6L^2} \times 1 = \frac{\pi}{6}$



2. Partition within a square duct:

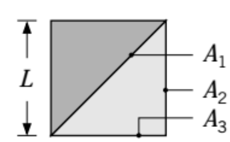
From summation rule, $F_{11} + F_{12} + F_{13} = 1$

where $F_{11} = 0$

By symmetry, $F_{12} = F_{13}$

Hence $F_{12} = 0.50$

By reciprocity, $F_{21} = \frac{A_1}{A_2} F_{12} = \frac{\sqrt{2}\, L}{L} \times 0.5 = 0.71$



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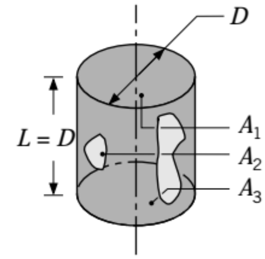
Find F_{12} and F_{21}?



3. Circular tube:

From Table 13.2 or Figure 13.5, with $(r_3/L) = 0.5$ and $(L/r_1) = 2$, $F_{13} = 0.172$

From summation rule, $F_{11} + F_{12} + F_{13} = 1$

or, with $F_{11} = 0$, $F_{12} = 1 - F_{13} = 0.828$

From reciprocity, $$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{\pi D^2/4}{\pi DL} \times 0.828 = 0.207$$

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