UNIT 4 POWER TRANSMISSION DEVICES (Part -A)

INTRODUCTION

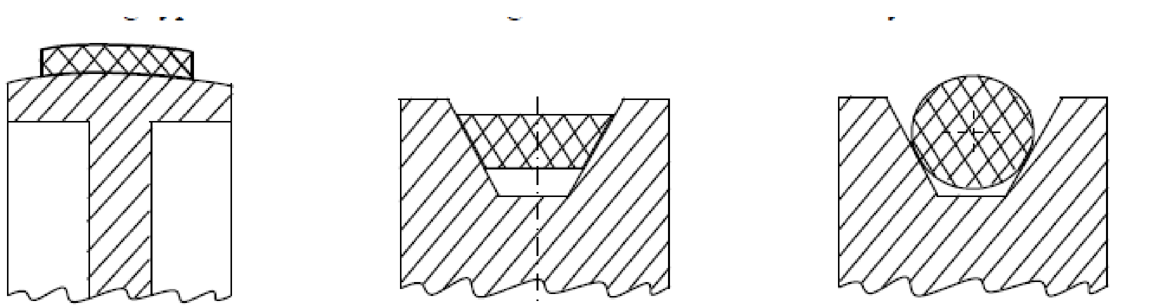
The power is transmitted from one shaft to the other by means of belts, chains and gears. The belts and ropes are flexible members which are used where distance between the two shafts is large. The chains also have flexibility but they are preferred for intermediate distances. The gears are used when the shafts are very close with each other. This type of drive is also called positive drive because there is no slip. If the distance is slightly larger, chain drive can be used for making it a positive drive. Belts and ropes transmit power due to the friction between the belt or rope and the pulley. There is a possibility of slip and creep and that is why, this drive is not a positive drive. A gear train is a combination of gears which are used for transmitting motion from one shaft to another.

POWER TRANSMISSION DEVICES

Power transmission devices are very commonly used to transmit power from one shaft to another. Belts, chains and gears are used for this purpose. When the distance between the shafts is large, belts or ropes are used and for intermediate distance chains can be used. For belt drive distance can be maximum but this should not be more than ten metres for good results. Gear drive is used for short distances.

Belts

In case of belts, friction between the belt and pulley is used to transmit power. In practice, there is always some amount of slip between belt and pulleys, therefore, exact velocity ratio cannot be obtained. That is why, belt drive is not a positive drive. Therefore, the belt drive is used where exact velocity ratio is not required.
The following types of belts shown in Figure 1 are most commonly used :



(a) Flat Belt and Pulley (b) V-belt and Pulley (c) Circular Belt or Rope Pulley

Figure 1 : Types of Belt and Pulley

The flat belt is rectangular in cross-section as shown in Figure 1(a). The pulley for this belt is slightly crowned to prevent slip of the belt to one side. It utilizes the friction between the flat surface of the belt and pulley.
The V-belt is trapezoidal in section as shown in Figure 1(b). It utilizes the force of friction between the inclined sides of the belt and pulley. They are preferred when distance is comparative shorter. Several V-belts can also be used together if power transmitted is more.
The circular belt or rope is circular in section as shown in Figure 1(c). Several ropes also can be used together to transmit more power.
The belt drives are of the following types :
(a) open belt drive, and

(b) cross belt drive.

Open Belt Drive

Open belt drive is used when sense of rotation of both the pulleys is same. It is desirable to keep the tight side of the belt on the lower side and slack side at the top to increase the angle of contact on the pulleys. This type of drive is shown in Figure 2.

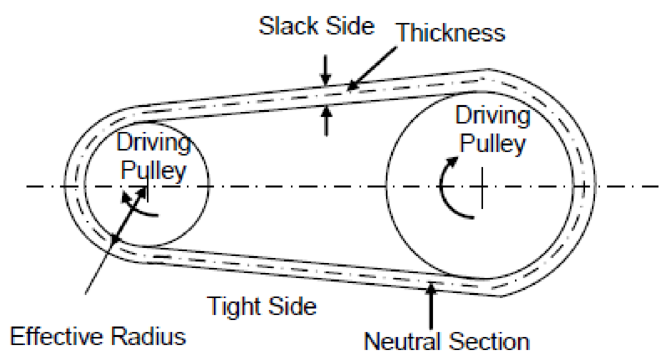


Figure 2 : Open Belt Derive

Cross Belt Drive

In case of cross belt drive, the pulleys rotate in the opposite direction. The angle of contact of belt on both the pulleys is equal. This drive is shown in Figure 3. As shown in the figure, the belt has to bend in two different planes. As a result of this, belt wears very fast and therefore, this type of drive is not preferred for power transmission. This can be used for transmission of speed at low power.

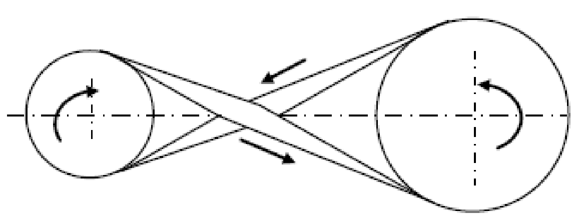


Figure 3 : Cross Belt Drive

Since power transmitted by a belt drive is due to the friction, belt drive is subjected to slip and creep.

Let d_1 and d_2 be the diameters of driving and driven pulleys, respectively. N_1 and N_2 be the corresponding speeds of driving and driven pulleys, respectively.

The velocity of the belt passing over the driver

$$V_1 = \frac{\pi d_1 N_1}{60}$$

If there is no slip between the belt and pulley

$$V_1 = V_2 = \frac{\pi d_2 N_2}{60}$$

or, $$\frac{\pi d_1 N_1}{60} = \frac{\pi d_2 N_2}{60}$$

or, $$\frac{N_1}{N_2} = \frac{d_2}{d_1}$$

If thickness of the belt is 't', and it is not negligible in comparison to the diameter,

$$\frac{N_1}{N_2} = \frac{d_2 + t}{d_1 + t}$$

Let there be total percentage slip 'S' in the belt drive which can be taken into account as follows :

$$V_2 = V_1 \left(1 - \frac{S}{100}\right)$$

or $$\frac{\pi d_2 N_2}{60} = \frac{\pi d_1 N_1}{60} \left(1 - \frac{S}{100}\right)$$

If the thickness of belt is also to be considered

or $$\frac{N_1}{N_2} = \frac{(d_2 + t)}{(d_1 + t)} \times \frac{1}{\left(1 - \frac{S}{100}\right)}$$

or, $$\frac{N_2}{N_1} = \frac{(d_1 + t)}{(d_2 + t)} \times \left(1 - \frac{S}{100}\right)$$

The belt moves from the tight side to the slack side and vice-versa, there is some loss of power because the length of belt continuously extends on tight side and contracts on loose side. Thus, there is relative motion between the belt and pulley due to body slip. This is known as creep.

POWER TRANSMISSION BY BELTS

Law of Belting

The law of belting states that the centre line of the belt as it approaches the pulley, must lie in plane perpendicular to the axis of the pulley in the mid plane of the pulley otherwise the belt will run off the pulley. However, the point at which the belt leaves the other pulley must lie in the plane of a pulley.

The Figure 4 below shows the belt drive in which two pulleys are at right angle to each other. It can be seen that the centre line of the belt approaching larger or smaller pulley lies in its plane. The point at which the belt leaves is contained in the plane of the other pulley.

If motion of the belt is reversed, the law of the belting will be violated. Therefore, motion is possible in one direction in case of non-parallel shafts as shown in Figure 4.

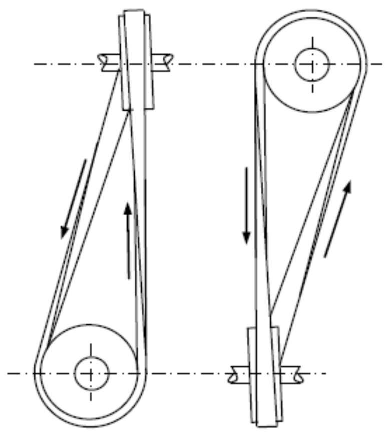


Figure 4 : Law of Belting

Length of the Belt

For any type of the belt drive it is always desirable to know the length of belt required. It will be required in the selection of the belt. The length can be determined by the geometric considerations. However, actual length is slightly shorter than the theoretically determined value.

Open Belt Drive

The open belt drive is shown in Figure 5. Let O_1 and O_2 be the pulley centers and *AB* and *CD* be the common tangents on the circles representing the two pulleys. The total length of the belt '*L*' is given by

$L = AB + \text{Arc } BHD + DC + \text{Arc } CGA$

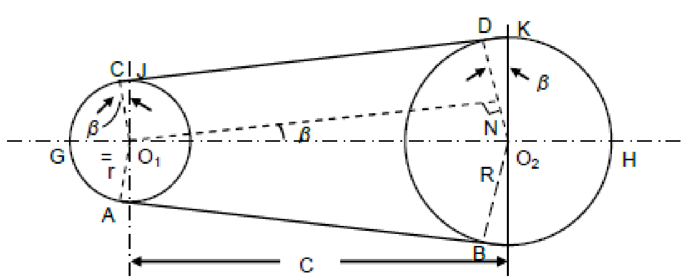


Figure 5 : Open Belt Drive

Let r be the radius of the smaller pulley,

R be the radius of the larger pulley,

C be the centre distance between the pulleys, and

β be the angle subtended by the tangents AB and CD with $O_1\, O_2$.

Draw $O_1\, N$ parallel to CD to meet $O_2 D$ at N.

By geometry, $\angle\, O_2\, O_1, N = \angle\, C\, O_1\, J = \angle\, D\, O_2\, K = \beta$

$$\text{Arc } BHD = (\pi + 2\beta)\, R,$$

$$\text{Arc } CGA = (\pi - 2\beta)\, r$$

$$AB = CD = O_1\, N = O_1\, O_2 \cos\beta = C\cos\beta$$

$$\sin\beta = \frac{R - r}{C}$$

or,
$$\beta = \sin^{-1}\frac{(R - r)}{C}$$

$$\cos\beta = \sqrt{1 - \sin^2\beta} \approx \left(1 - \frac{1}{2}\sin^2\beta\right)$$

$\therefore$
$$L = (\pi + 2\beta)\, R + (\pi - 2\beta)\, r + 2C\left(1 - \frac{1}{2}\sin^2\beta\right)$$

For small value of β; $\beta = \dfrac{(R - r)}{C}$, the approximate lengths

$$L = \pi\,(R + r) + 2\,(R - r)\,\frac{(R - r)}{C} + 2C\left[1 - \frac{1}{2}\left(\frac{R - r}{C}\right)^2\right]$$

$$= \pi\,(R + r) + \frac{(R - r)^2}{C} + 2C\left[1 - \frac{1}{2}\left(\frac{R - r}{C}\right)^2\right]$$

Crossed-Belt Drive

The crossed-belt drive is shown in Figure 6. Draw $O_1 N$ parallel to the line *CD* which meets extended O_2 *D* at *N*. By geometry

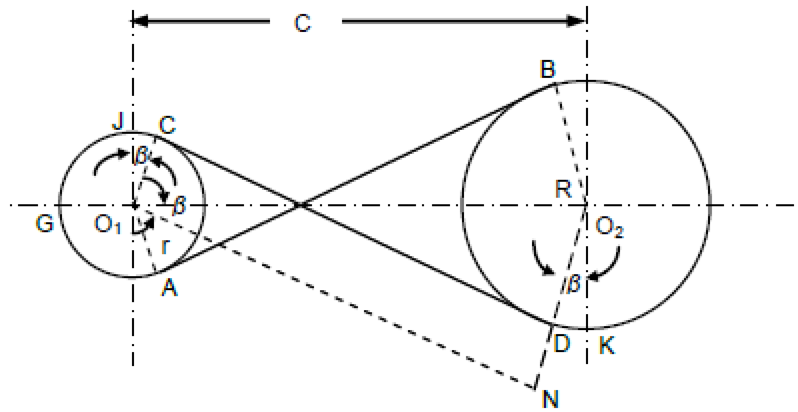


Figure 6 : Cross Belt Drive

$$\angle CO_1 J = \angle DO_2 K = \angle O_2 O_1 N$$

$$L = \text{Arc } AGC + AB + \text{Arc } BKD + CD$$

$$\text{Arc } AGC = r\,(\pi + 2\beta), \text{ and Arc } BKD = (\pi + 2\beta)\,R$$

$$\sin\beta = \frac{R+r}{C} \quad \text{or} \quad \beta = \sin^{-1}\frac{(R+r)}{C}$$

For small value of β

$$\beta \approx \frac{R+r}{C}$$

$$\cos\beta = \sqrt{1+\sin^2\beta} \approx \left(1 - \frac{1}{2}\sin^2\beta\right) = \left[1 - \frac{1}{2}\frac{(R+r)^2}{C^2}\right]$$

$$L = r\,(\pi + 2\beta) + 2C\cos\beta + R\,(\pi + 2\beta)$$

$$= (\pi + 2\beta)\,(R + r) + 2C\cos\beta$$

For approximate length

$$L = \pi\,(R + r) + 2\,\frac{(R+r)^2}{C} + 2C\left[1 - \frac{1}{2}\frac{(R+r)^2}{C^2}\right]$$

$$= \pi\,(R + r) + \frac{(R+r)^2}{C} + 2C$$

Ratio of Tensions

The belt drive is used to transmit power from one shaft to the another. Due to the friction between the pulley and the belt one side of the belt becomes tight side and other becomes slack side. We have to first determine ratio of tensions.

Flat Belt

Let tension on the tight side be 'T_1' and the tension on the slack side be 'T_2'. Let 'θ' be the angle of lap and let 'μ' be the coefficient of friction between the belt and the pulley. Consider an infinitesimal length of the belt *PQ* which subtend an angle $\delta\theta$ at the centre of the pulley. Let '*R*' be the reaction between the element and the pulley. Let '*T*' be tension on the slack side of the element, i.e. at point *P* and let '$(T + \delta T)$' be the tension on the tight side of the element.

The tensions *T* and $(T + \delta T)$ shall be acting tangential to the pulley and thereby normal to the radii *OP* and *OQ*. The friction force shall be equal to 'μR' and its action will be to prevent slipping of the belt. The friction force will act tangentially to the pulley at the point *S*.

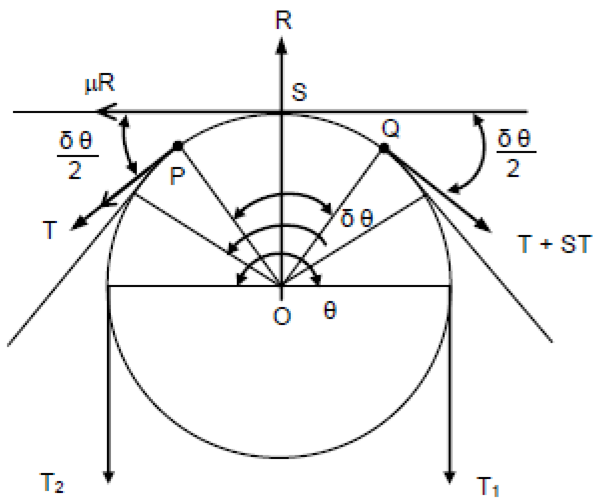


Figure 7 : Ratio of Tensions in Flat Belt

Considering equilibrium of the element at *S* and equating it to zero.

Resolving all the forces in the tangential direction

$$\mu R + T \cos\frac{\delta\theta}{2} - (T + \delta T)\cos\frac{\delta\theta}{2} = 0$$

or, $$\mu R = \delta T \cos\frac{\delta\theta}{2}$$

Resolving all the forces in the radial direction at S and equating it to zero.

$$R - T \sin \frac{\delta\theta}{2} - (T + \delta T) \sin \frac{\delta\theta}{2} = 0$$

or, $$R = (2T + \delta T) \sin \frac{\delta\theta}{2}$$

Since $\delta\theta$ is very small, taking limits

$\therefore$ $$\cos \frac{\delta\theta}{2} \approx 1 \text{ and } \sin \frac{\delta\theta}{2} = \frac{\delta\theta}{2}$$

$\therefore$ $$R = (2T + \delta T) \frac{\delta\theta}{2} = T\,\delta\theta + \delta T \frac{\delta\theta}{2}$$

Neglecting the product of the two infinitesimal quantities $\left(\delta T \frac{\delta\theta}{2}\right)$ which is negligible in comparison to other quantities :

$\therefore$ $$R \approx T\,\delta\theta$$

Substituting the value of R and $\cos \frac{\delta\theta}{2} \approx 1$ in Eq. (3.4), we get

$$\mu T\,\delta\theta = \delta T$$

or, $$\frac{\delta T}{T} = \mu \delta\theta$$

Taking limits on both sides as $\delta\,\theta \to 0$

$$\frac{dT}{T} = \mu\, d\,\theta$$

Integrating between limits, it becomes

$$\int_{T_2}^{T_1} \frac{dT}{T} = \int_0^{\theta} \mu\, d\theta$$

or, $$\ln \frac{T_1}{T_2} = \mu\theta$$

or, $$\frac{T_1}{T_2} = e^{\mu\theta} \qquad \ldots$$

Power Transmitted by Belt Drive

The power transmitted by the belt depends on the tension on the two sides and the belt speed.
Let T_1 be the tension on the tight side in '*N*'
T_2 be the tension on the slack side in '*N*', and
V be the speed of the belt in m/sec.
Then power transmitted by the belt is given by:

$$\text{Power } P = (T_1 - T_2)\, V \text{ Watt}$$

$$= \frac{(T_1 - T_2)\, V}{1000} \text{ kW} \qquad . .$$

or,

$$P = \frac{T_1 \left(1 - \frac{T_2}{T_1}\right) V}{1000} \text{ kW}$$

If belt is on the point of slipping.

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

$$\therefore \qquad P = \frac{T_1 (1 - e^{-\mu\theta})\, V}{1000} \text{ kW} \qquad . .$$

The maximum tension T_1 depends on the capacity of the belt to withstand force. If allowable stress in the belt is 'σ_t' in 'Pa', i.e. N/m^2, then

$$T_1 = (\sigma_t \times t \times b) \text{ N} \qquad . . .$$

where t is thickness of the belt in 'm' and b is width of the belt also in m.

The above equations can also be used to determine 'b' for given power and speed.

Tension due to Centrifugal Forces

The belt has mass and as it rotates along with the pulley it is subjected to centrifugal forces. If we assume that no power is being transmitted and pulleys are rotating, the centrifugal force will tend to pull the belt as shown in Figure 8(b) and, thereby, a tension in the belt called centrifugal tension will be introduced.

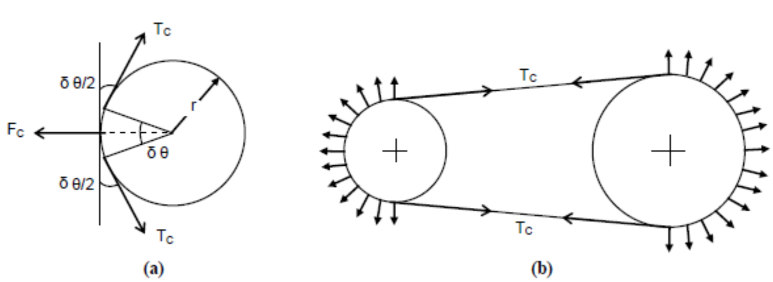


Figure 8 : Tension due to Centrifugal Foces

Let 'T_C' be the centrifugal tension due to centrifugal force.

Let us consider a small element which subtends an angle $\delta\theta$ at the centre of the pulley.

Let 'm' be the mass of the belt per unit length of the belt in 'kg/m'.

The centrifugal force 'F_c' on the element will be given by

$$F_C = (r\ \delta\theta\ m) \times \frac{V^2}{r}$$

where V is speed of the belt in m/sec. and r is the radius of pulley in 'm'.

Resolving the forces on the element normal to the tangent

$$F_C - 2T_C \sin\frac{\delta\theta}{2} = 0$$

Since $\delta\theta$ is very small.

$$\therefore \qquad \sin\frac{\delta\theta}{2} \approx \frac{\delta\theta}{2}$$

or, $$F_C - 2T_C \frac{\delta\theta}{2} = 0$$

or, $$F_C = T_C\ \delta\theta$$

Substituting for F_C

$$\frac{m\,V^2}{r}\, r\ \delta\theta = T_C\ \delta\theta$$

or, $$T_C = m\,V^2$$

Therefore, considering the effect of the centrifugal tension, the belt tension on the tight side when power is transmitted is given by

Tension of tight side $T_t = T_1 + T_C$ and tension on the slack side $T_s = T_2 + T_C$.

The centrifugal tension has an effect on the power transmitted because maximum tension can be only T_t which is

$$T_t = \sigma_t \times t \times b$$

$$\therefore \qquad T_1 = \sigma_t \times t \times b - m\,V^2$$

Initial Tension in belt

When a belt is mounted on the pulley some amount of initial tension say 'T_0' is provided in the belt, otherwise power transmission is not possible because a loose belt cannot sustain difference in the tension and no power can be transmitted.

When the drive is stationary the total tension on both sides will be '2 T_0'.

When belt drive is transmitting power the total tension on both sides will be ($T_1 + T_2$), where T_1 is tension on tight side, and T_2 is tension on the slack side.

If effect of centrifugal tension is neglected.

$$2T_0 = T_1 + T_2$$

or, $$T_0 = \frac{T_1 + T_2}{2}$$

If effect of centrifugal tension is considered, then

$$T_0 = T_t + T_s = T_1 + T_2 + 2T_C$$

or, $$T_0 = \frac{T_1 + T_2}{2} + T_C$$

Maximum Power Transmitted

The power transmitted depends on the tension 'T_1', angle of lap θ, coefficient of friction 'μ' and belt speed 'V'. For a given belt drive, the maximum tension (T_t), angle of lap and coefficient of friction shall remain constant provided that

(a) the tension on tight side, i.e. maximum tension should be equal to the maximum permissible value for the belt, and

(b) the belt should be on the point of slipping.

Therefore, Power $P = T_1 (1 - e^{-\mu\theta})$ V

Since, $$T_1 = T_t + T_c$$

or, $$P = (T_t - T_c)(1 - e^{-\mu\theta}) V$$

or, $$P = (T_t - m V^2)(1 - e^{-\mu\theta}) V$$

For maximum power transmitted

∴ $$\frac{dP}{dV} = (T_t - 3m V^2)(1 - e^{-\mu\theta})$$

or, $$T_t - 3m V^2 = 0$$

or, $$T_t - 3T_c = 0$$

or, $$T_c = \frac{T_t}{3}$$

or, $$m V^2 = \frac{T_t}{3}$$